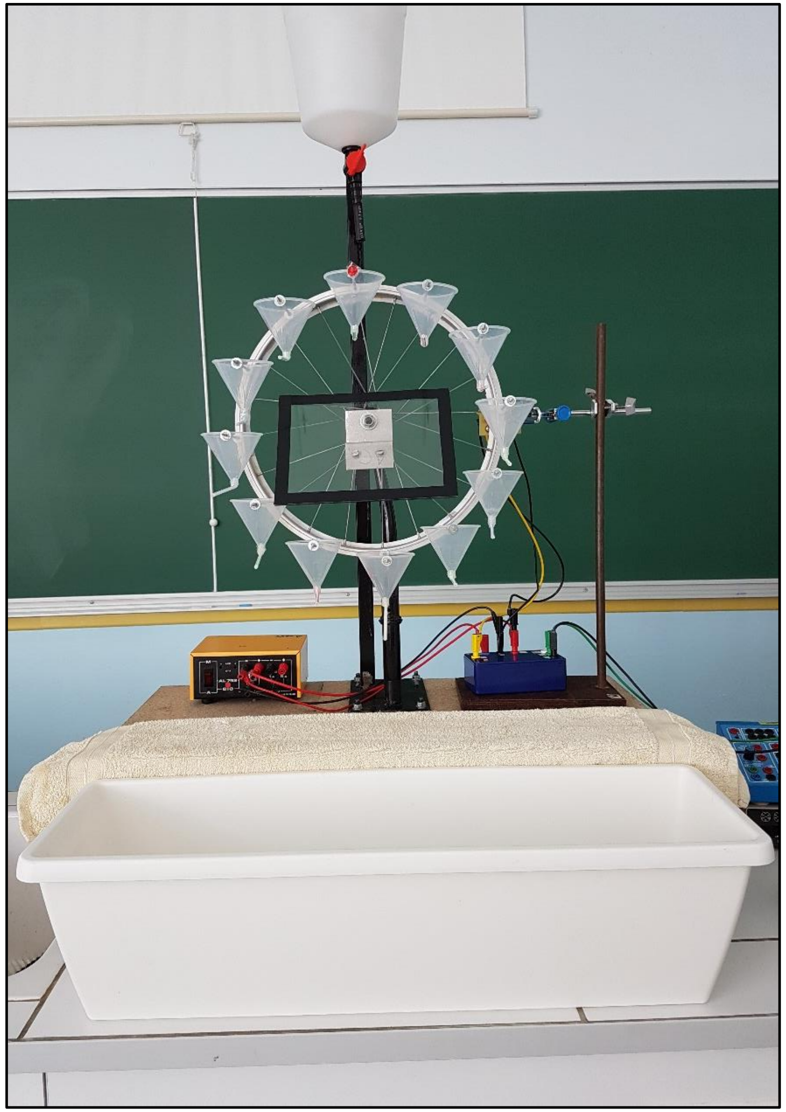
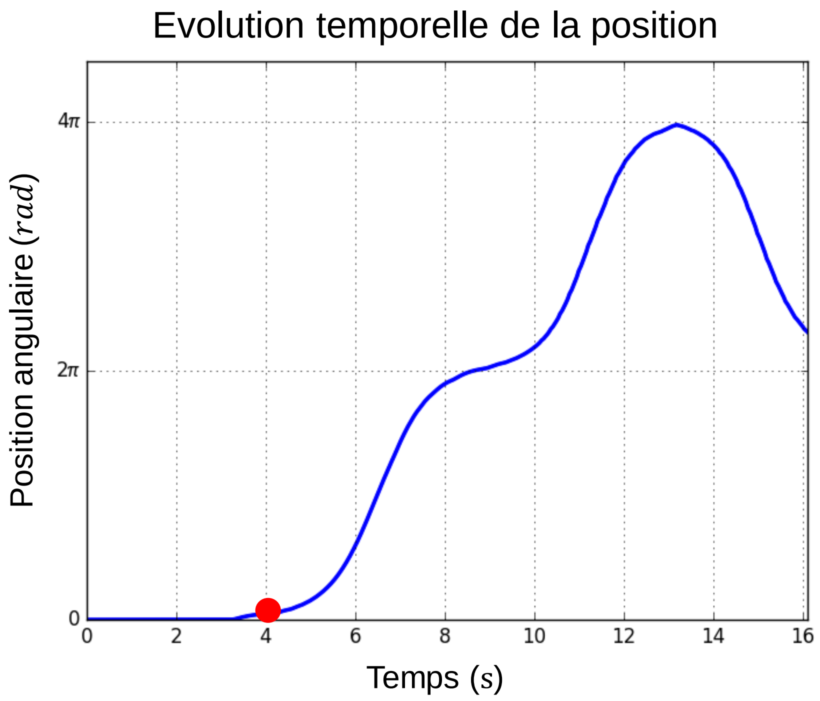
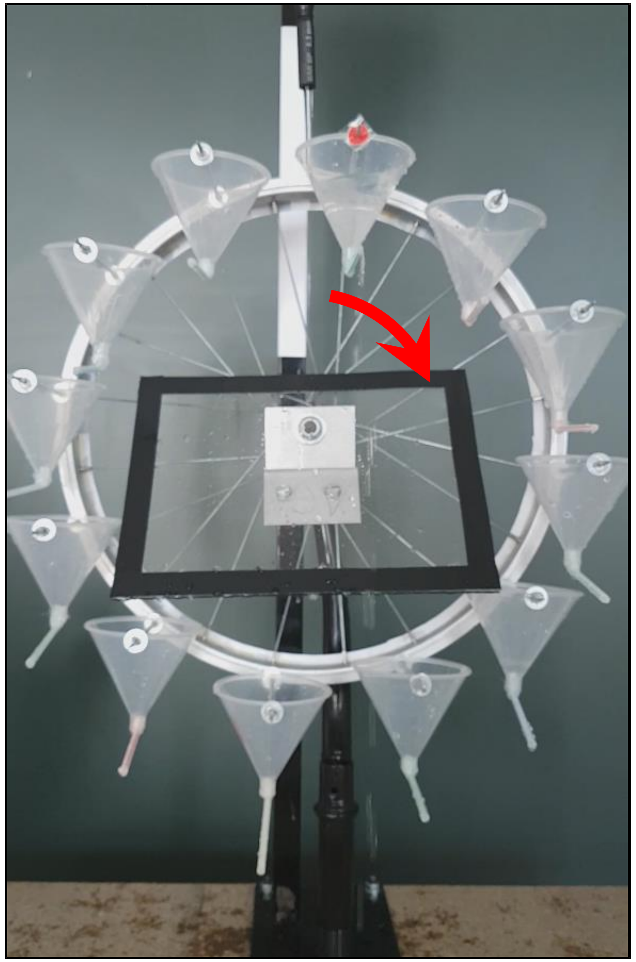
Comment pouvons-nous appréhender les écarts pour expliquer nos différences de comportements au cours de ces expériences ?

I. Présentation du système
II. Étude du système
III. L’effet papillon



Animation roue :

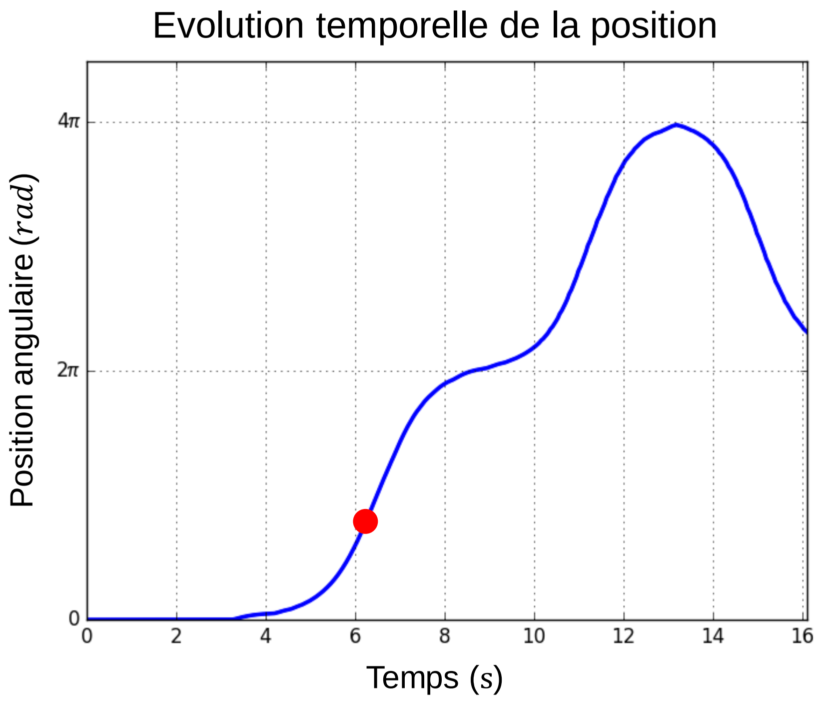


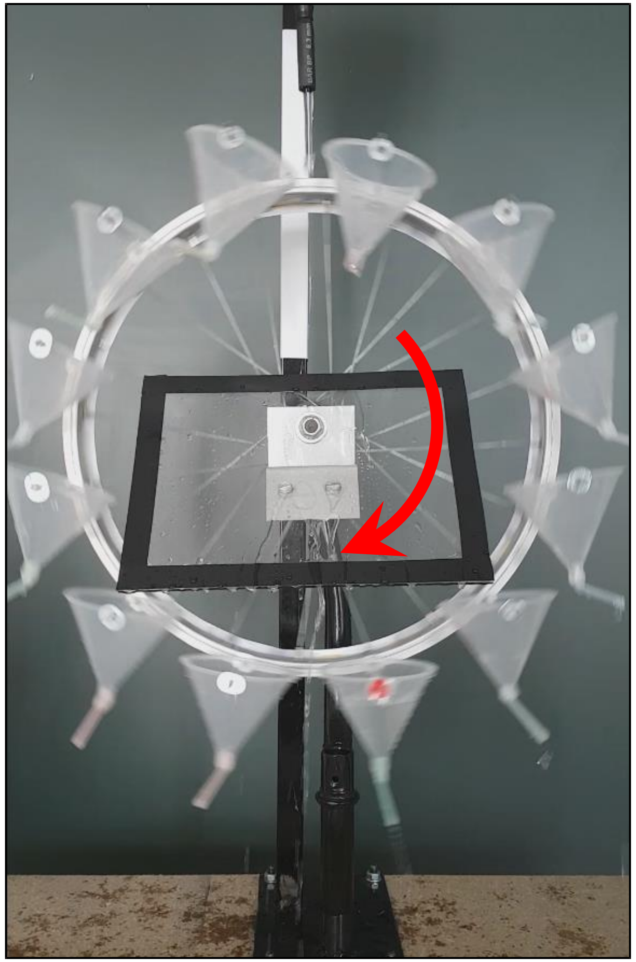


t= 4 s

$$\boxed{\begin{aligned}\boldsymbol{\theta_0} &= \mathbf{0,5°}\\ \boldsymbol{D_{eau}} &= \mathbf{5,9}\boldsymbol{g.s^{-1}}\end{aligned}}$$

Animation roue :

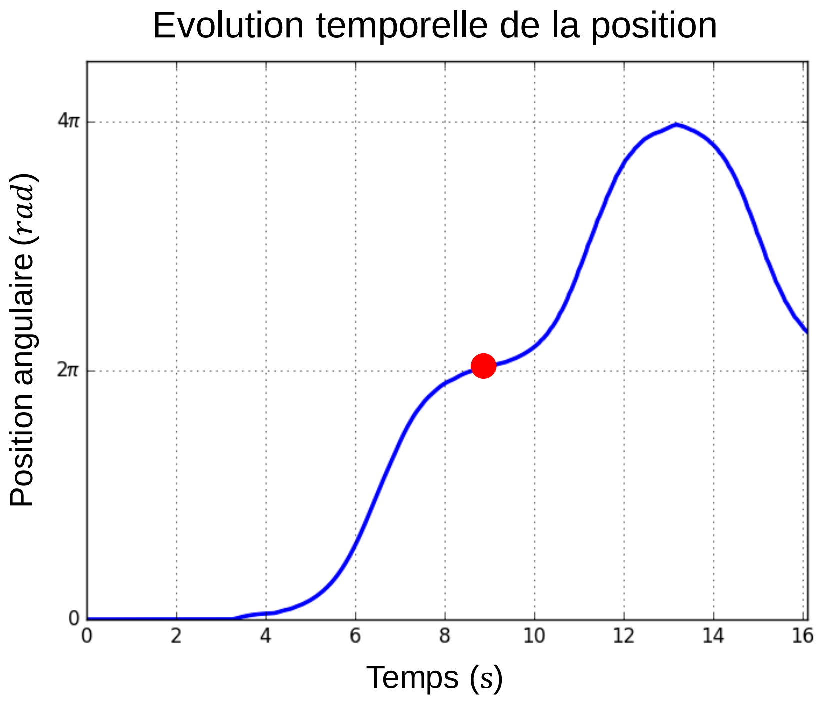


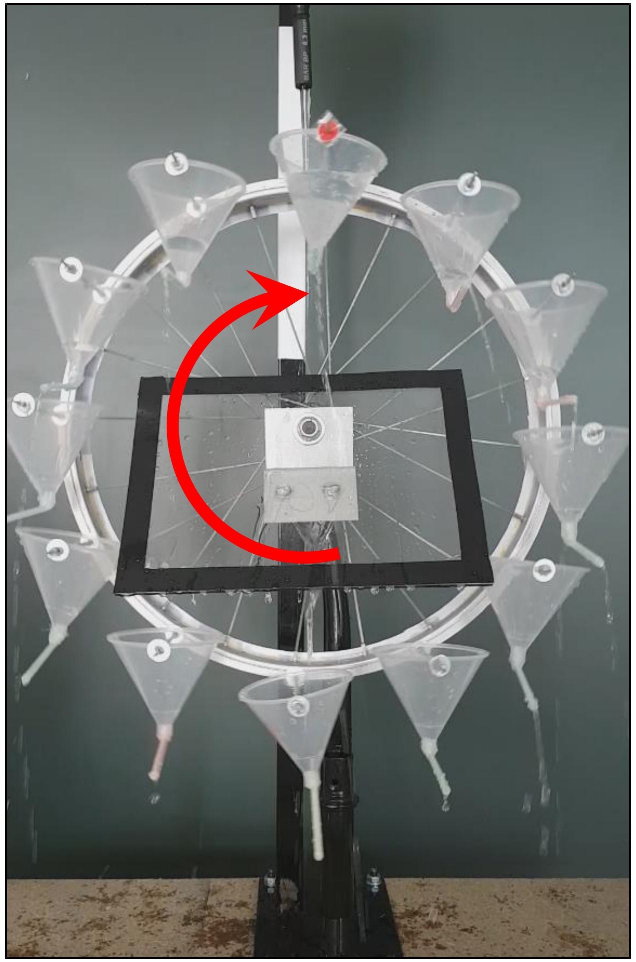


$$\boxed{\begin{array}{l}\boldsymbol{\theta_0 = 0,5°} \\ \boldsymbol{D_{eau} = 5,9 g.s^{-1}}\end{array}}$$

t= 6,3 s

Animation roue :

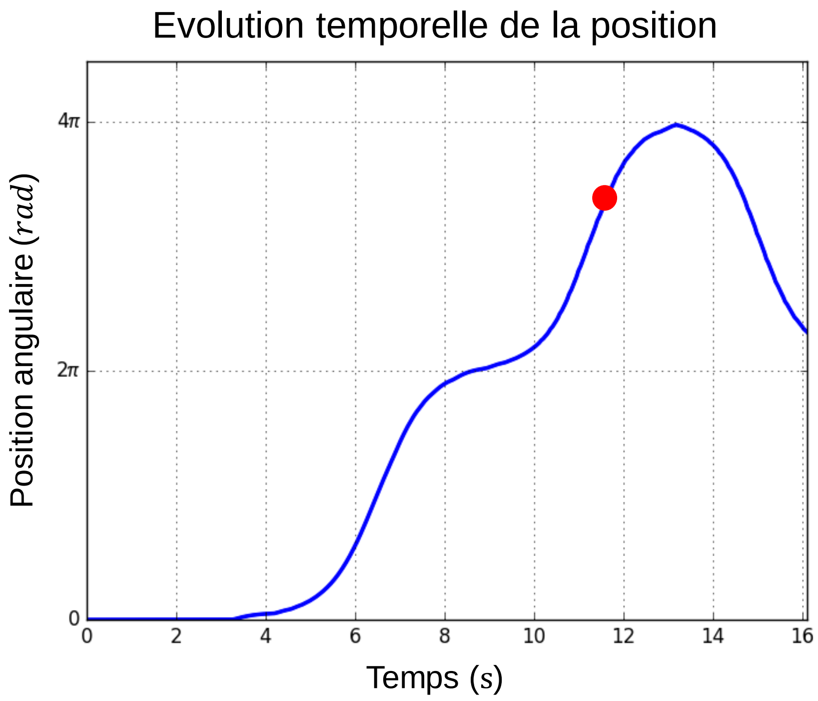


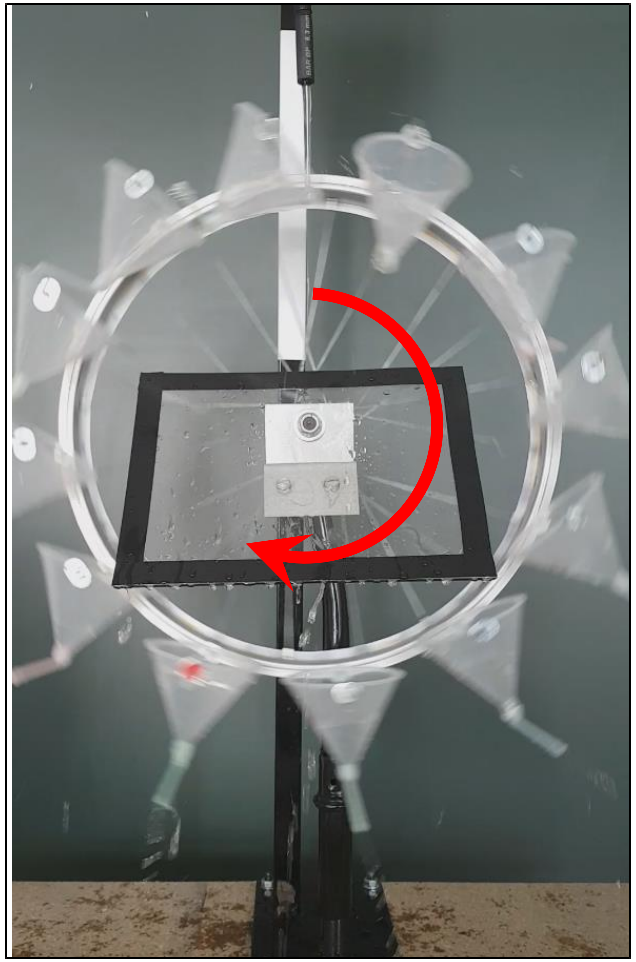


t= 9 s

$$\boxed{\begin{array}{l}\boldsymbol{\theta_0 = 0,5°}\\ \boldsymbol{D_{eau} = 5,9 g.s^{-1}}\end{array}}$$

Animation roue :

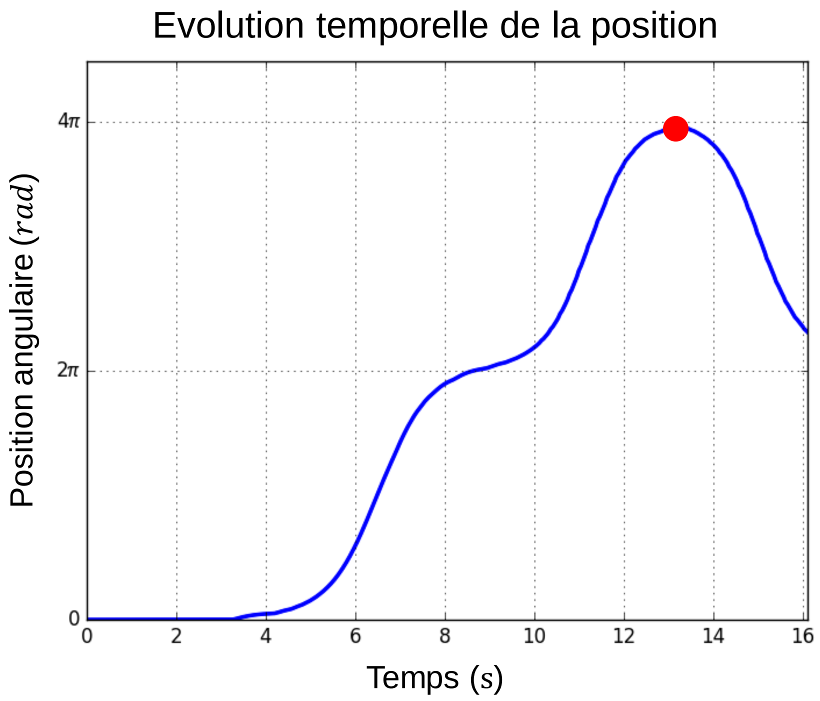


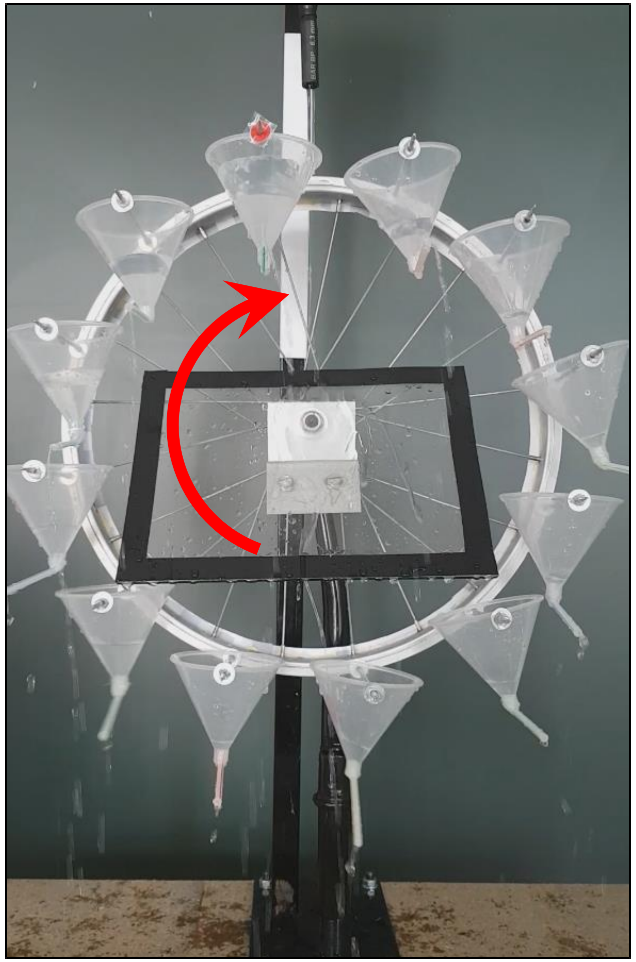


t= 11,5 s

$$\boxed{\begin{array}{l}\boldsymbol{\theta_0 = 0,5°} \\ \boldsymbol{D_{eau} = 5,9 g.s^{-1}}\end{array}}$$

Animation roue :

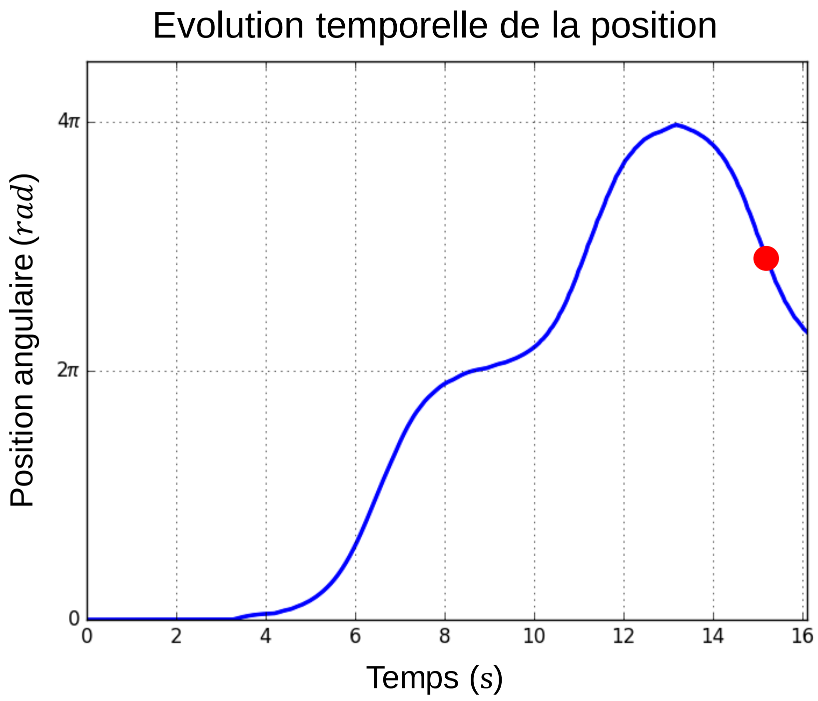


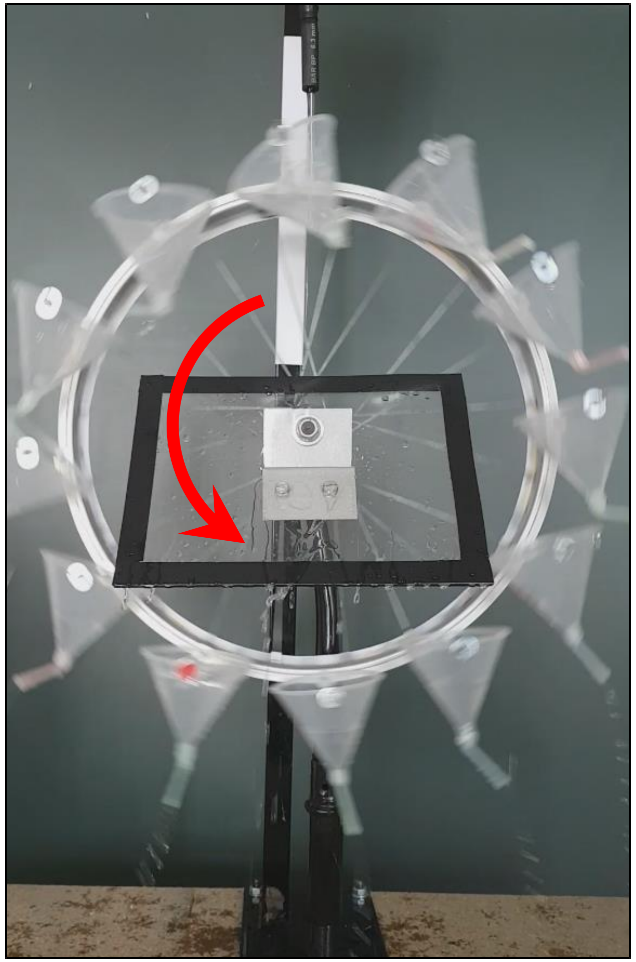


$$\boxed{\begin{aligned}\theta_0 &= 0,5° \\ D_{eau} &= 5,9 g.s^{-1}\end{aligned}}$$

t= 10 s

Animation roue :



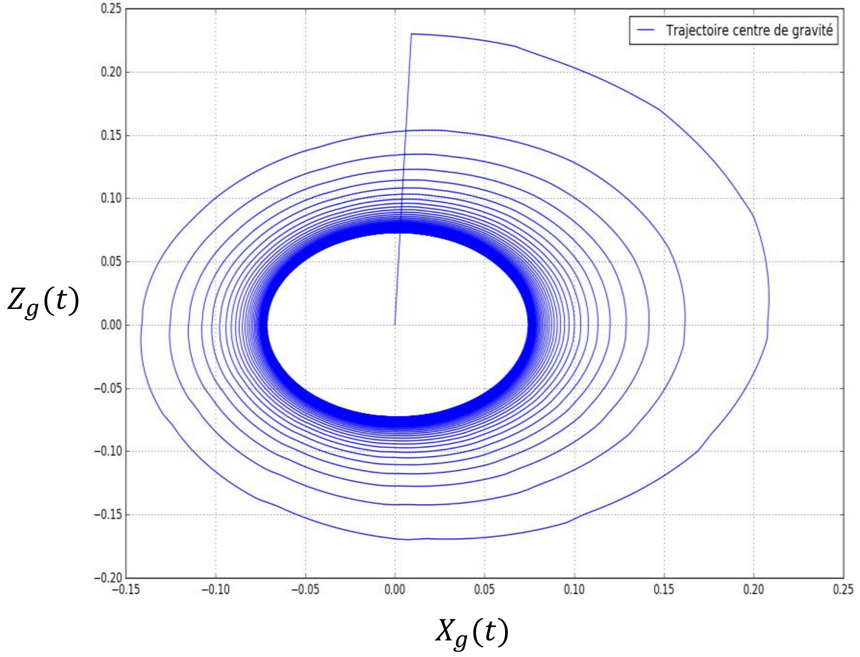


$$\boxed{\begin{array}{l}\boldsymbol{\theta_0 = 0,5°} \\ \boldsymbol{D_{eau} = 5,9g.s^{-1}}\end{array}}$$

t= 15 s

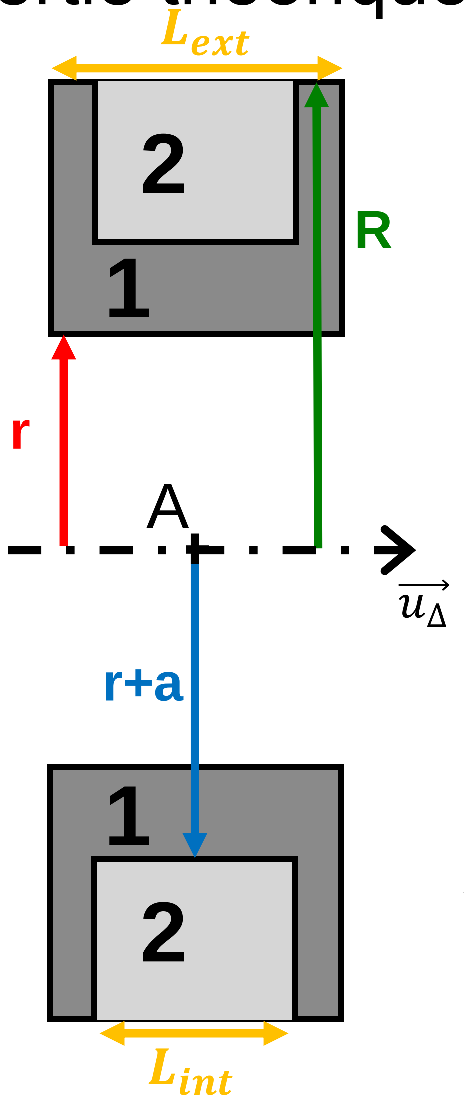
La modélisation :

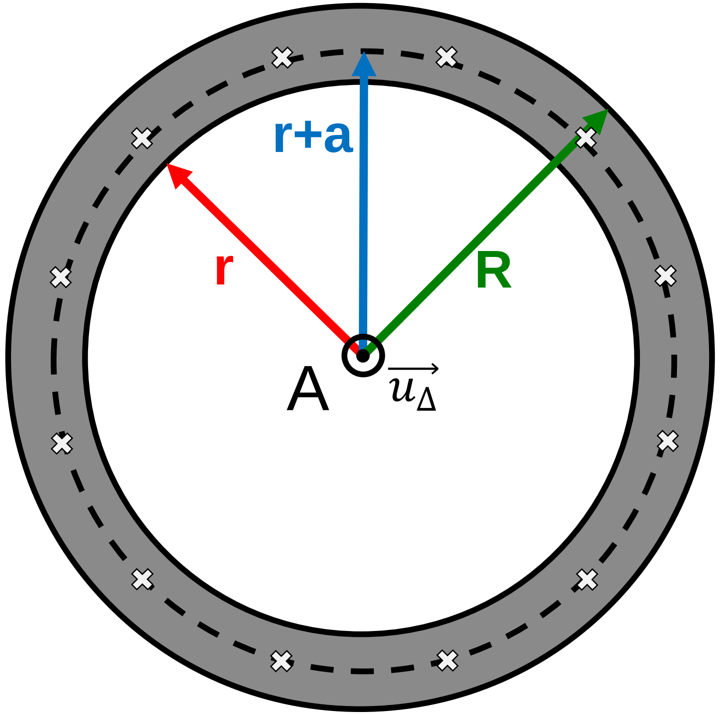
Evolution temporelle du centre de gravité

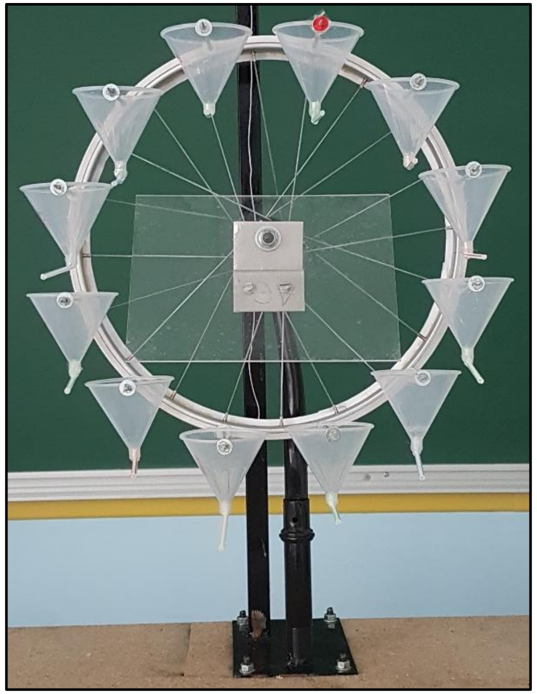


- Le moment d'inertie de notre roue de Lorenz
- Les frottements secs
- Le coefficient de frottement visqueux

Le moment d'inertie théorique :

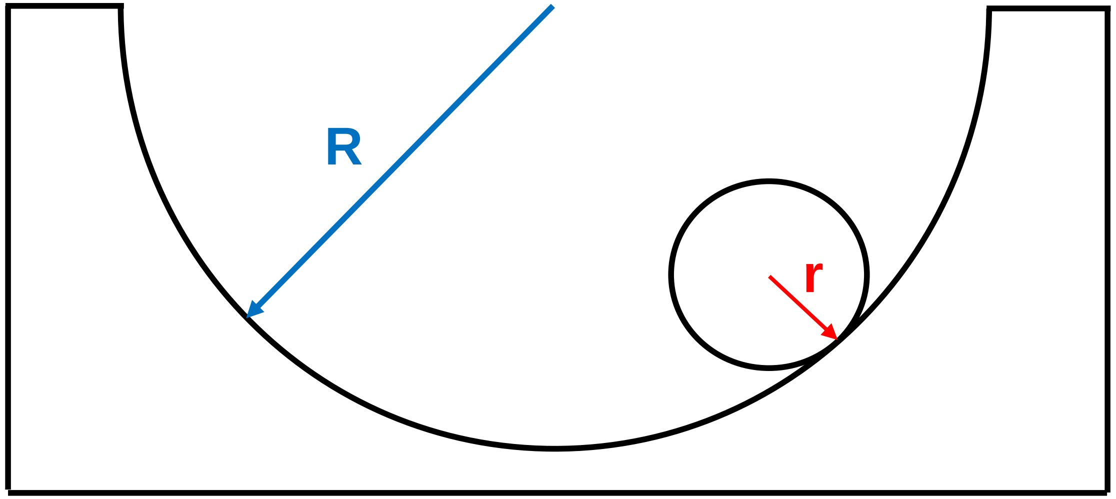


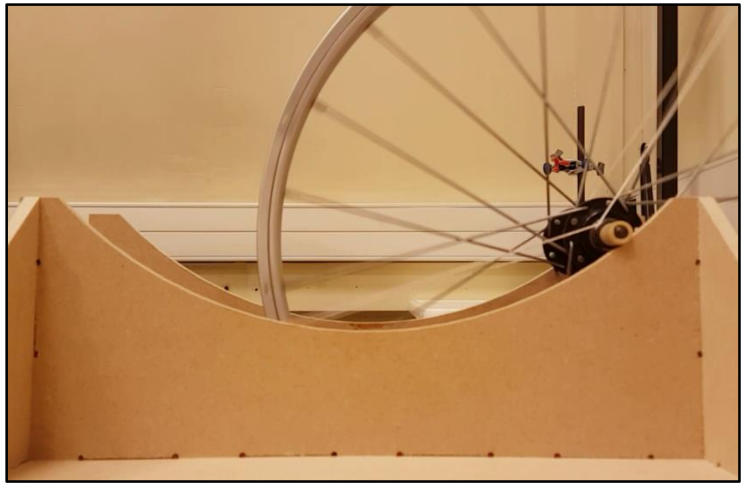




$$J_{th} = I^{1}_{(A,\overrightarrow{u_\Delta})} + I^{2}_{(A,\overrightarrow{u_\Delta})} + 12 \times I^{cône}_{(A,\overrightarrow{u_\Delta})} = 2{,}1.10^{-2} kg.m^2$$

Le moment d'inertie expérimental :

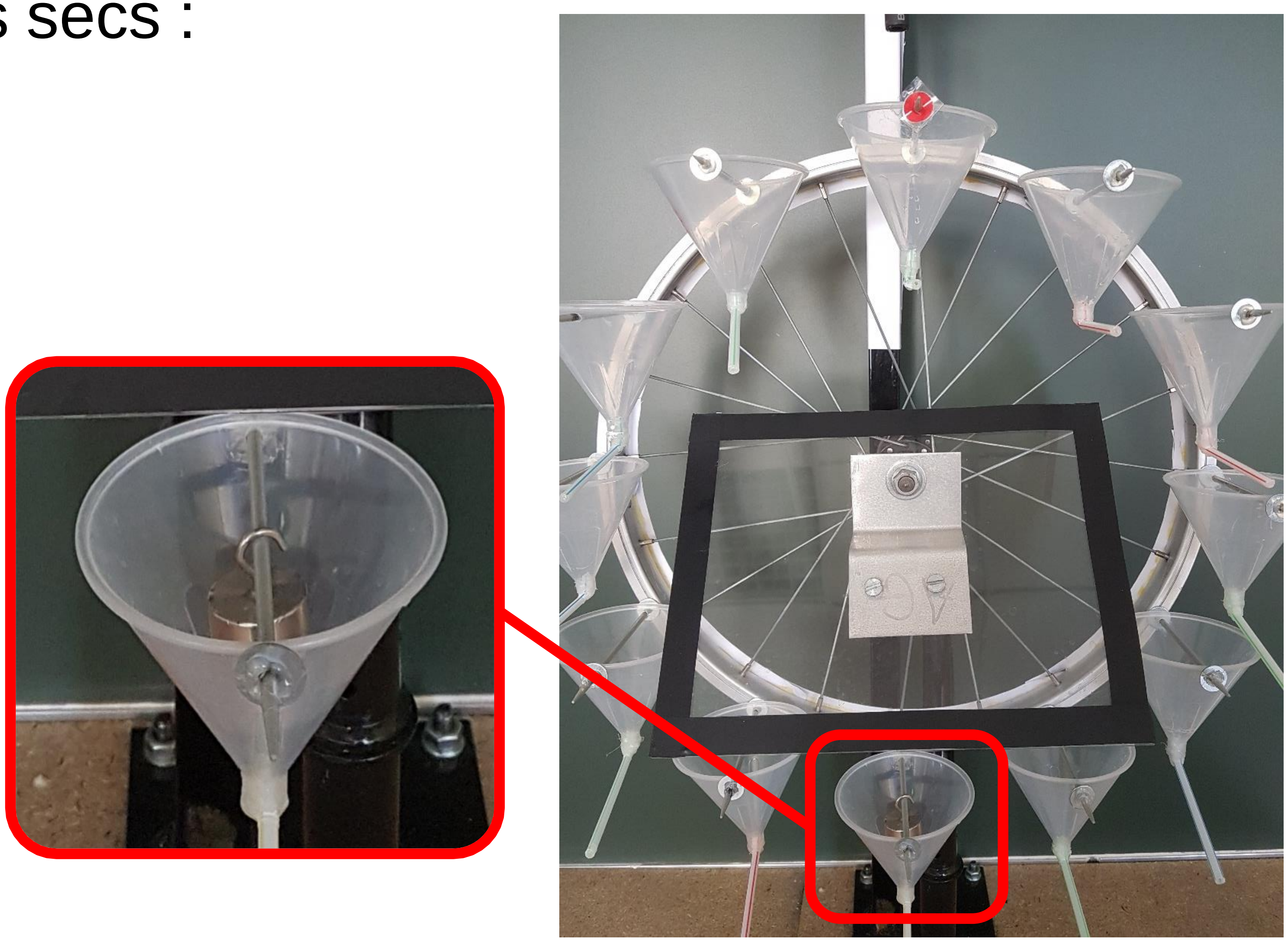




$T = 21{,}5\ s$

$$\begin{cases} J_{jante} = \dfrac{T^2}{4\pi^2}\dfrac{mgr^2}{R-r} - mr^2 \\ J_{cônes} = 12 \times I^{cône}_{(A,\overrightarrow{u_\Delta})} \end{cases} \Longrightarrow \begin{matrix} J_{exp} = 2{,}4.10^{-2} kg.m^2 \\ (J_{th} = 2{,}1.10^{-2} kg.m^2) \end{matrix}$$

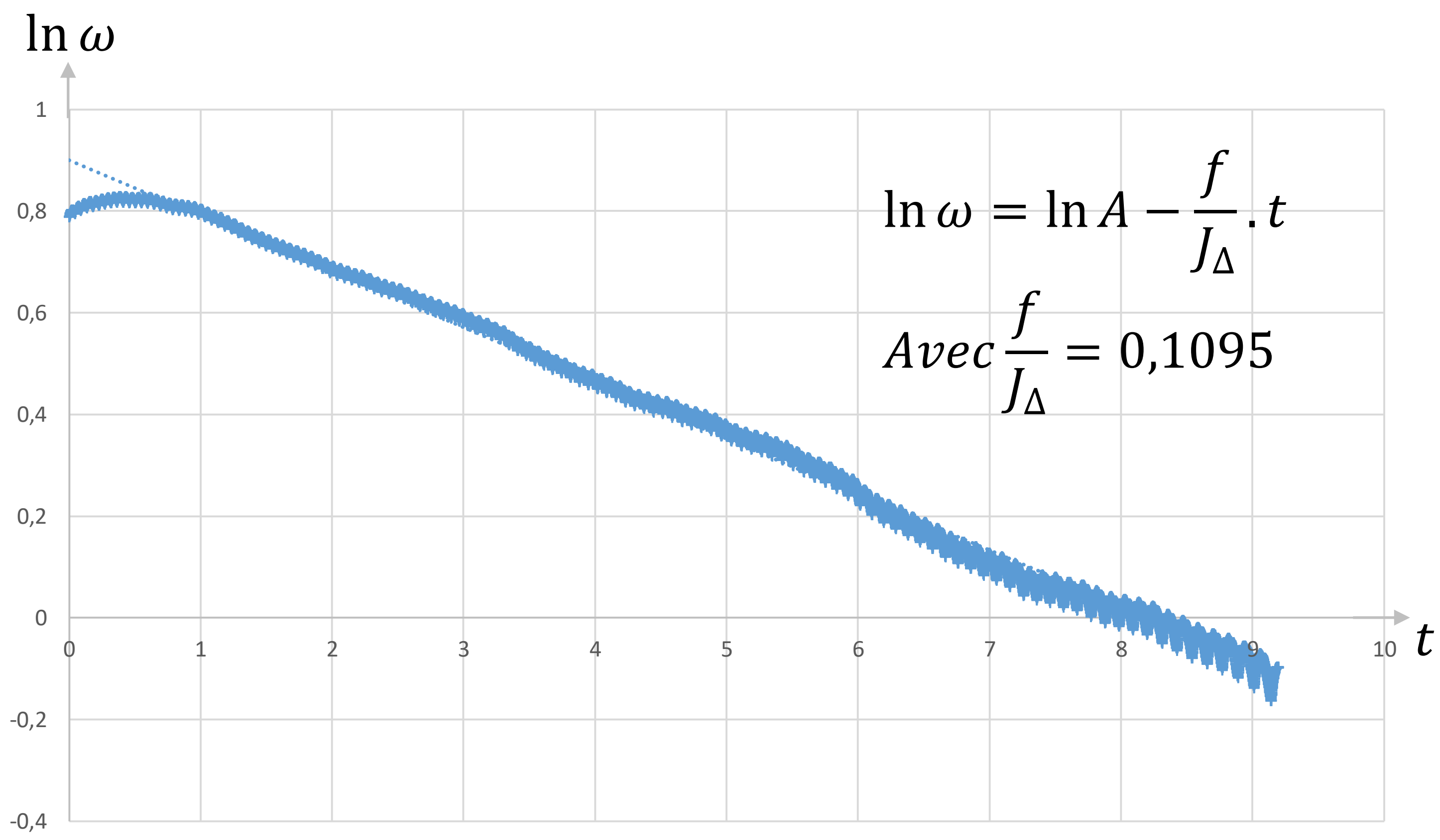
Les frottements secs :



Les frottements fluides :

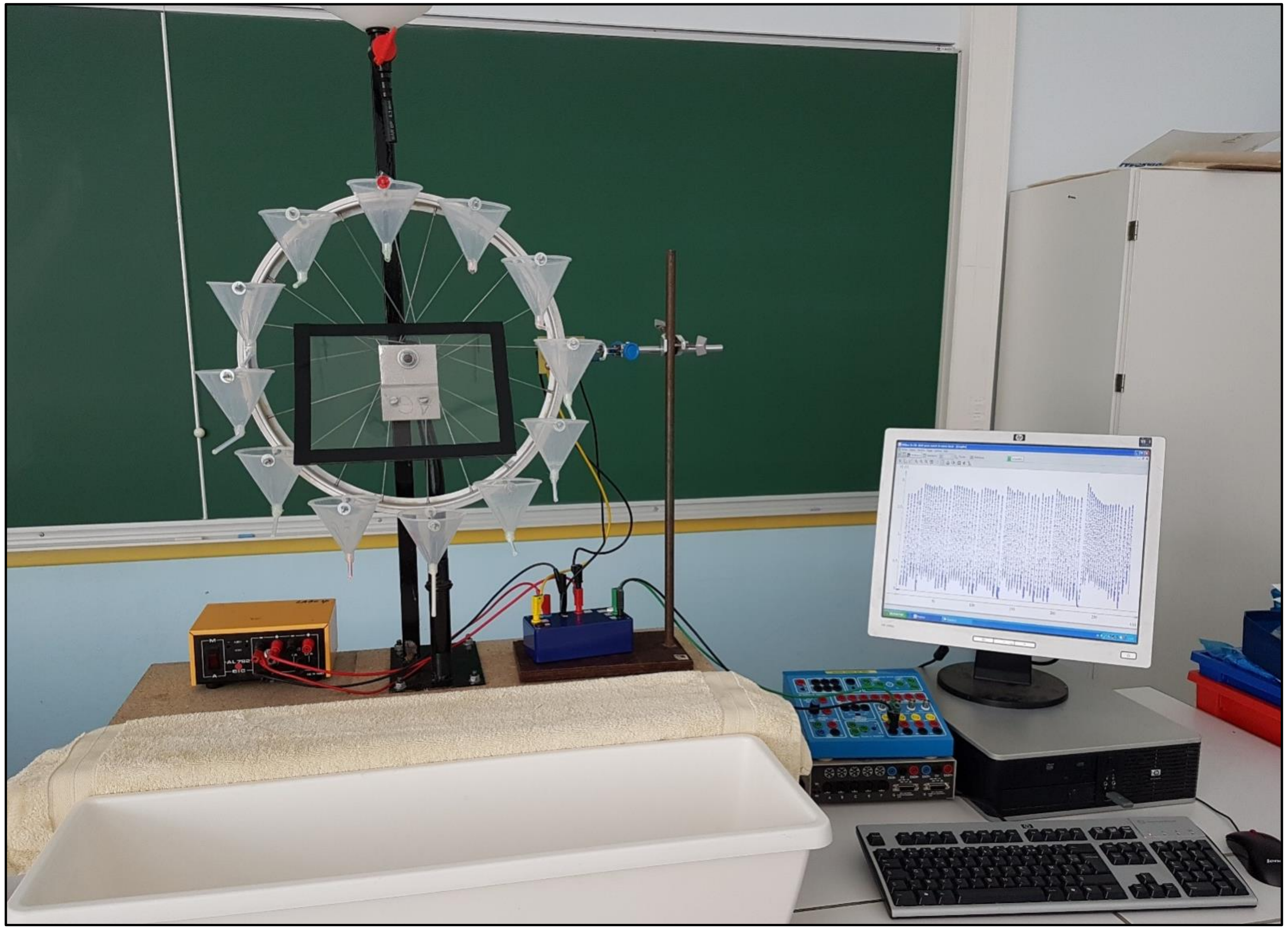
Equation du mouvement :

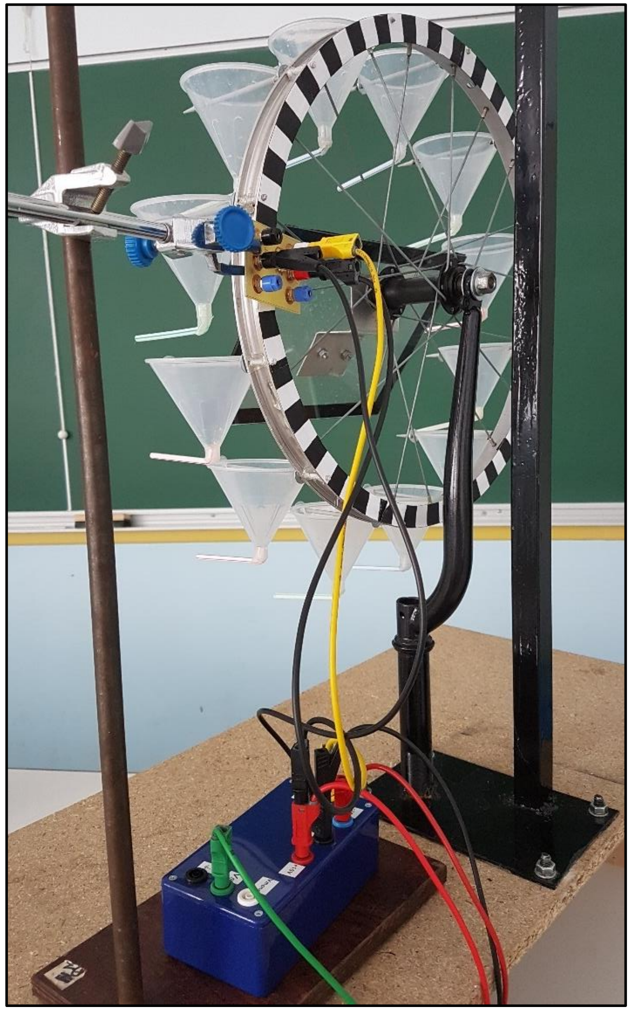
$$J_\Delta \frac{d\omega}{dt} = -f.\omega(t) \implies \omega(t) = A.e^{\frac{-f.t}{J_\Delta}}$$



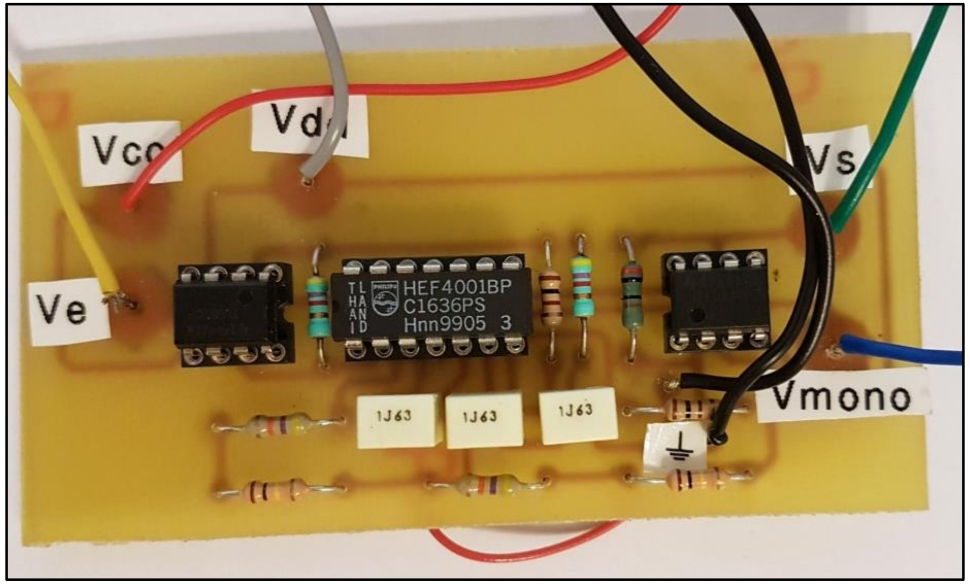
$$Donc, f = 2{,}7.10^{-3}\ N.m/(rad.s^{-1})$$

Installation :

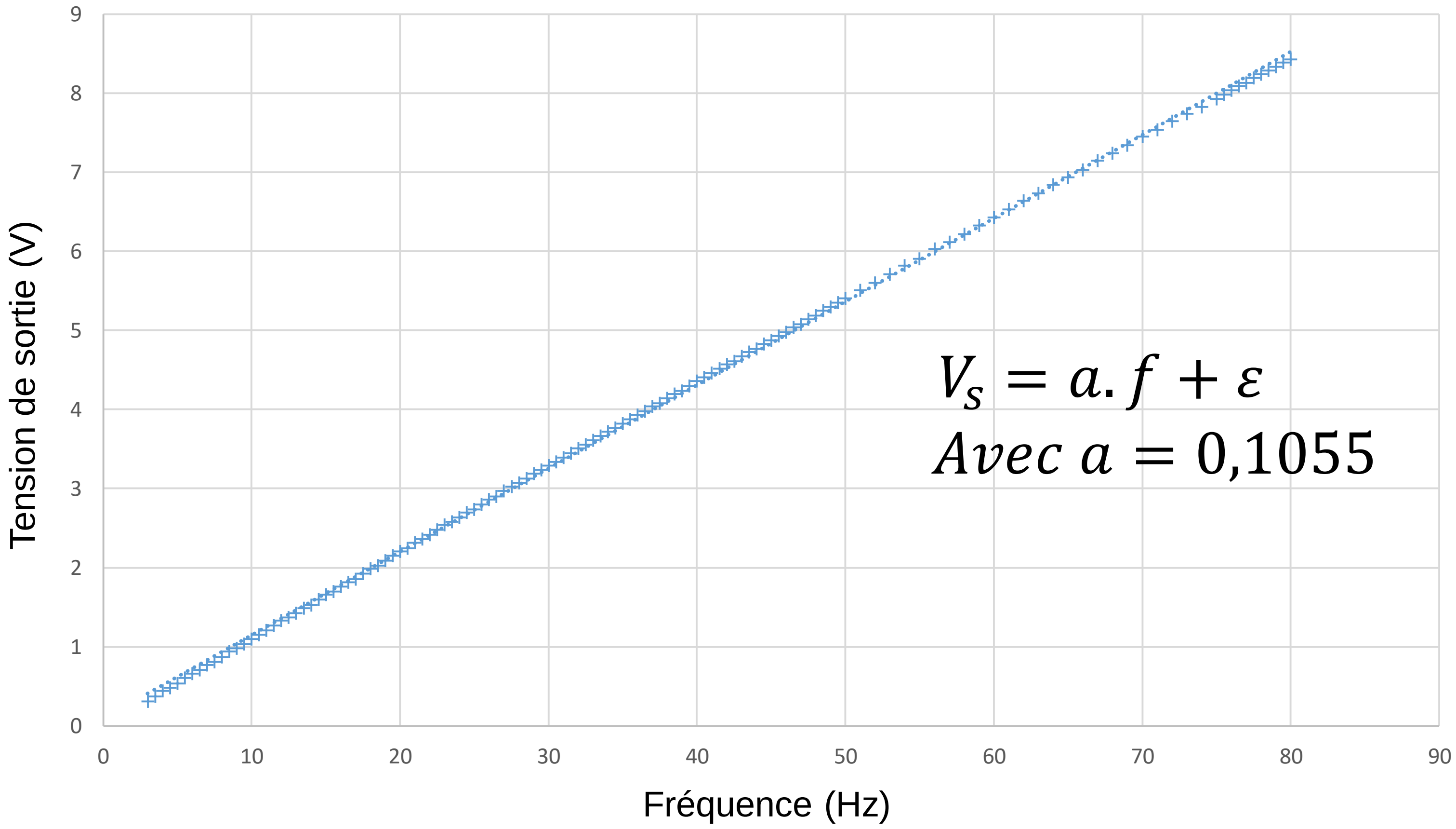




Capteur de vitesse :



$$\omega \longrightarrow \boxed{\frac{36 \cdot a}{2\pi}} \longrightarrow V_s$$



Traitement des données :

Expérience

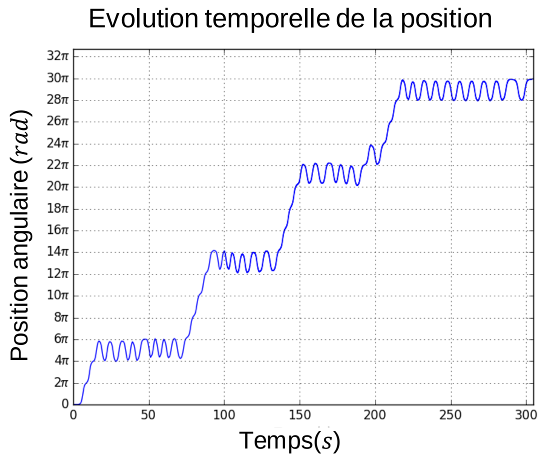
- Tableau de valeurs : $V_s = f(t)$

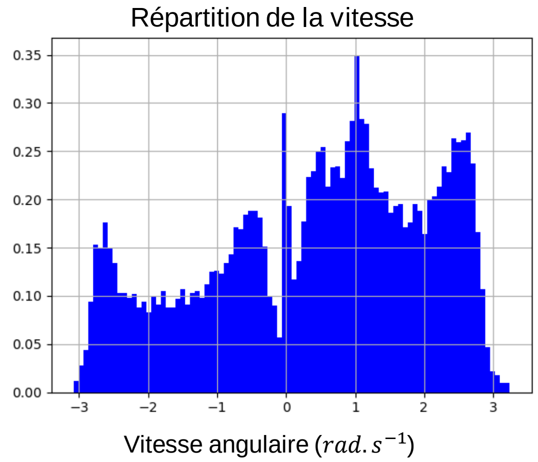
Programme Python

- Manipulation de liste
- Intégration de la vitesse → Position

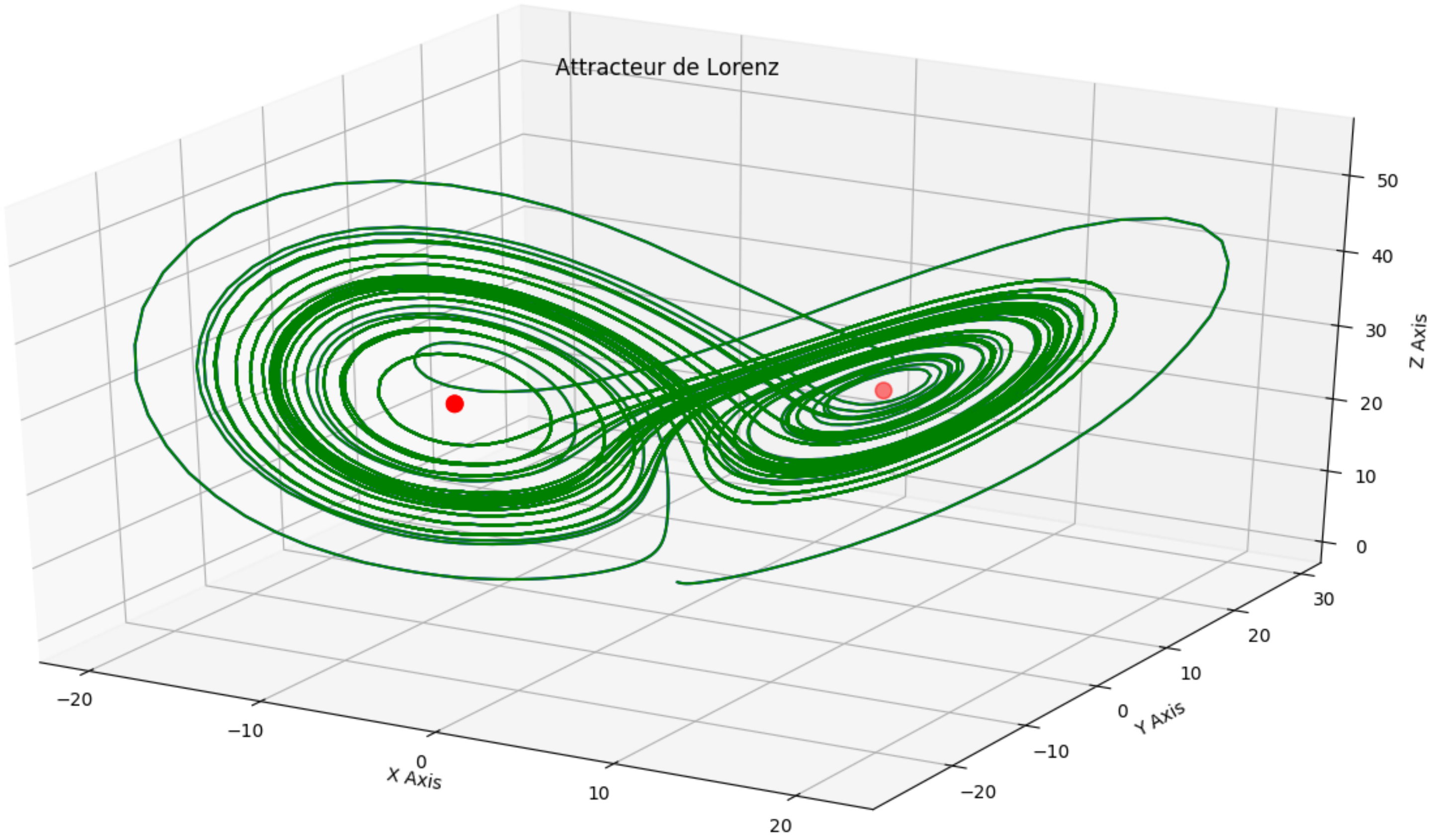
Visualisation des courbes

- Vitesse, position
- Portait de phase
- Histogramme de répartition de la vitesse





L’attracteur de Lorenz :

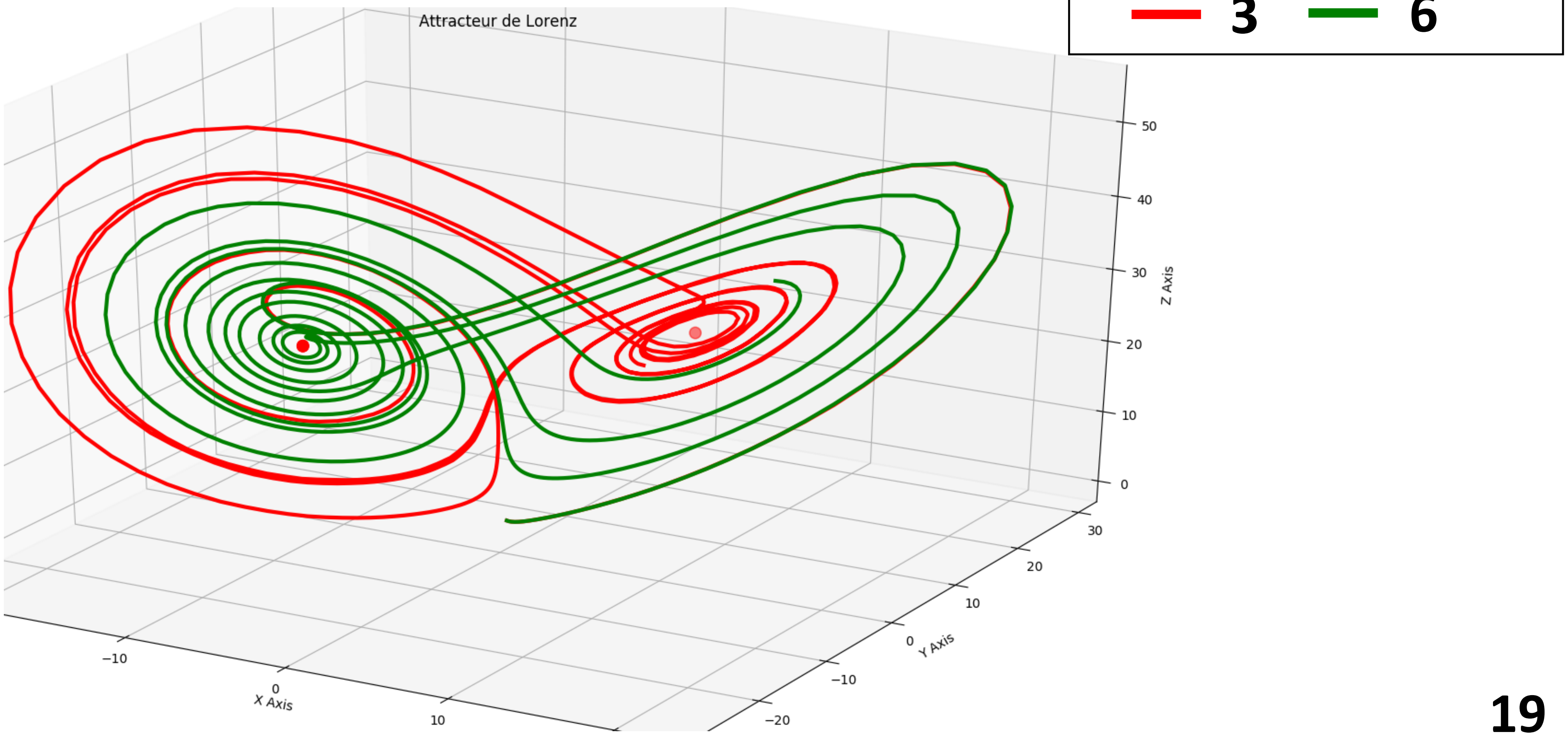


$$\begin{cases} \dfrac{dx}{dt} = \sigma \cdot \big(y(t) - x(t)\big) \\ \dfrac{dy}{dt} = \rho \cdot x(t) - y(t) - x(t) \cdot z(t) \\ \dfrac{dz}{dt} = x(t) \cdot y(t) - \beta \cdot z(t) \end{cases}$$

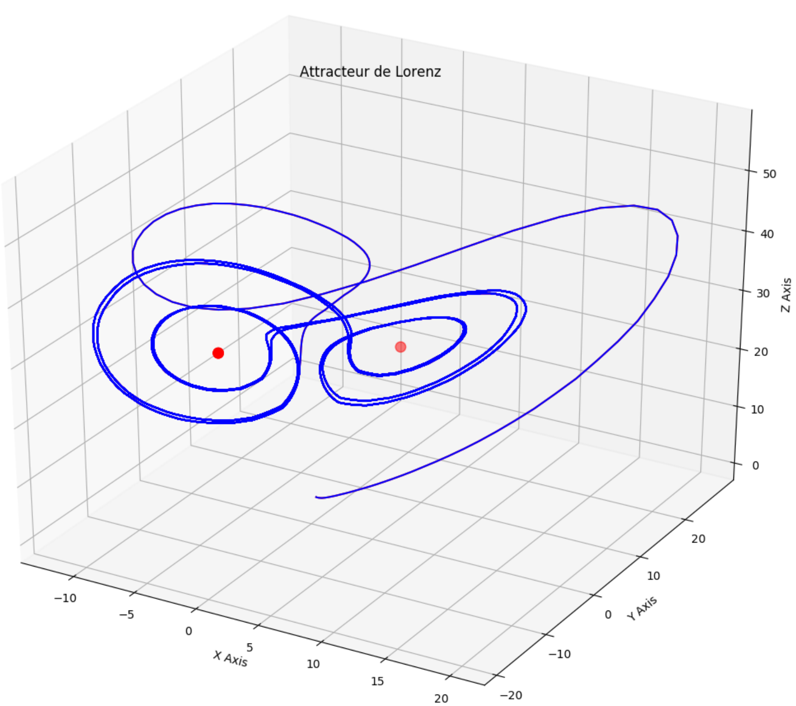
Chiffres significatifs :

$$x_0 = 0$$
$$y_0 = 1$$
$$z_0 = 1,05$$
$$\sigma = 10$$
$$\rho = 28$$
$$\beta = 2,667$$

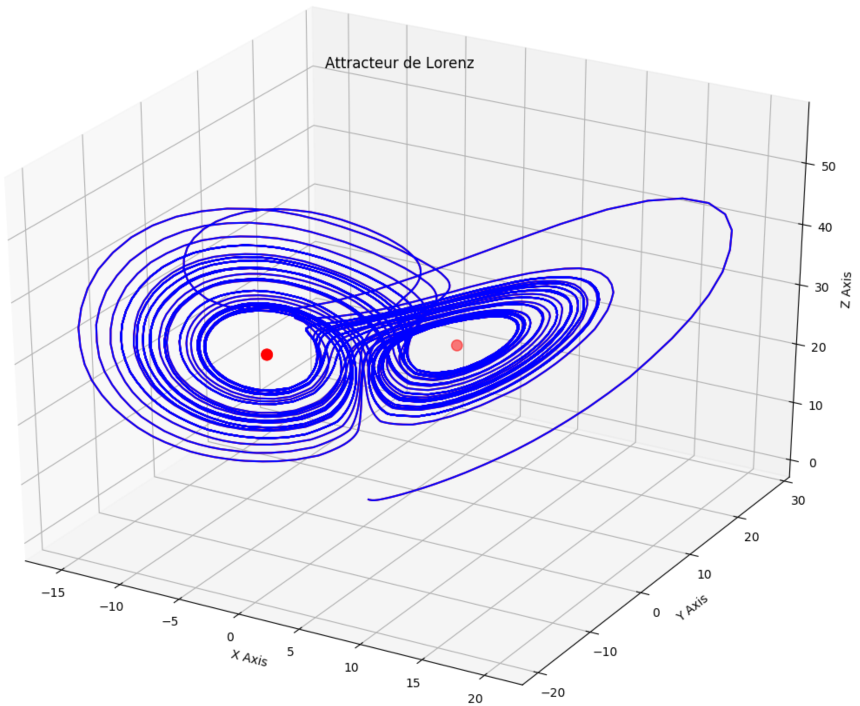
$$dt = 0,01$$
$$1000\ points$$



Conditions initiales :



z_0 =1,05

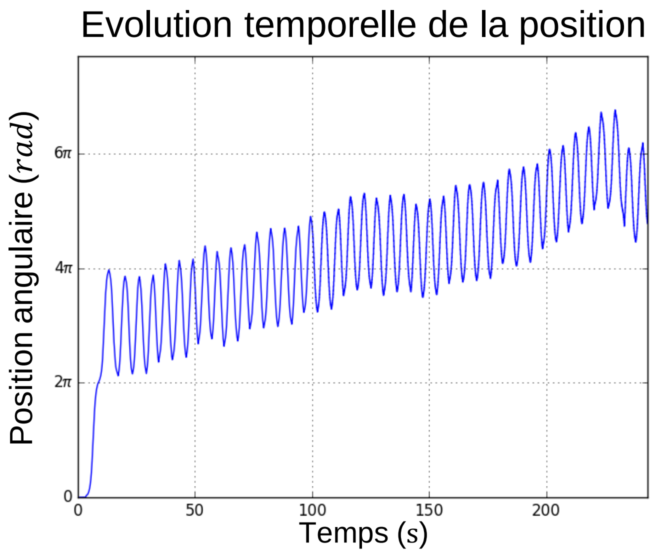


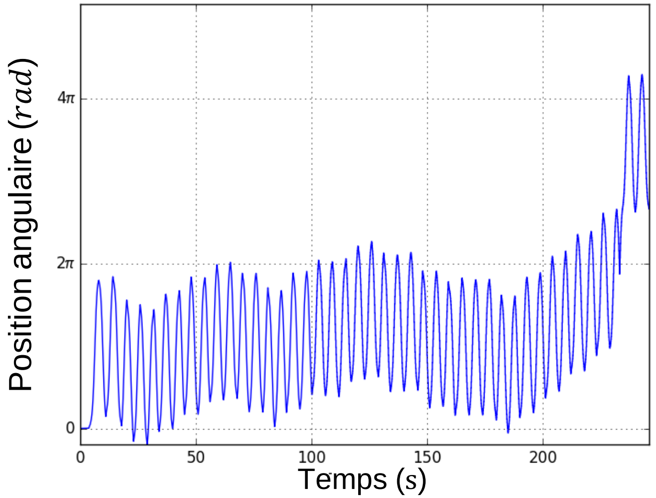
z_0 =1

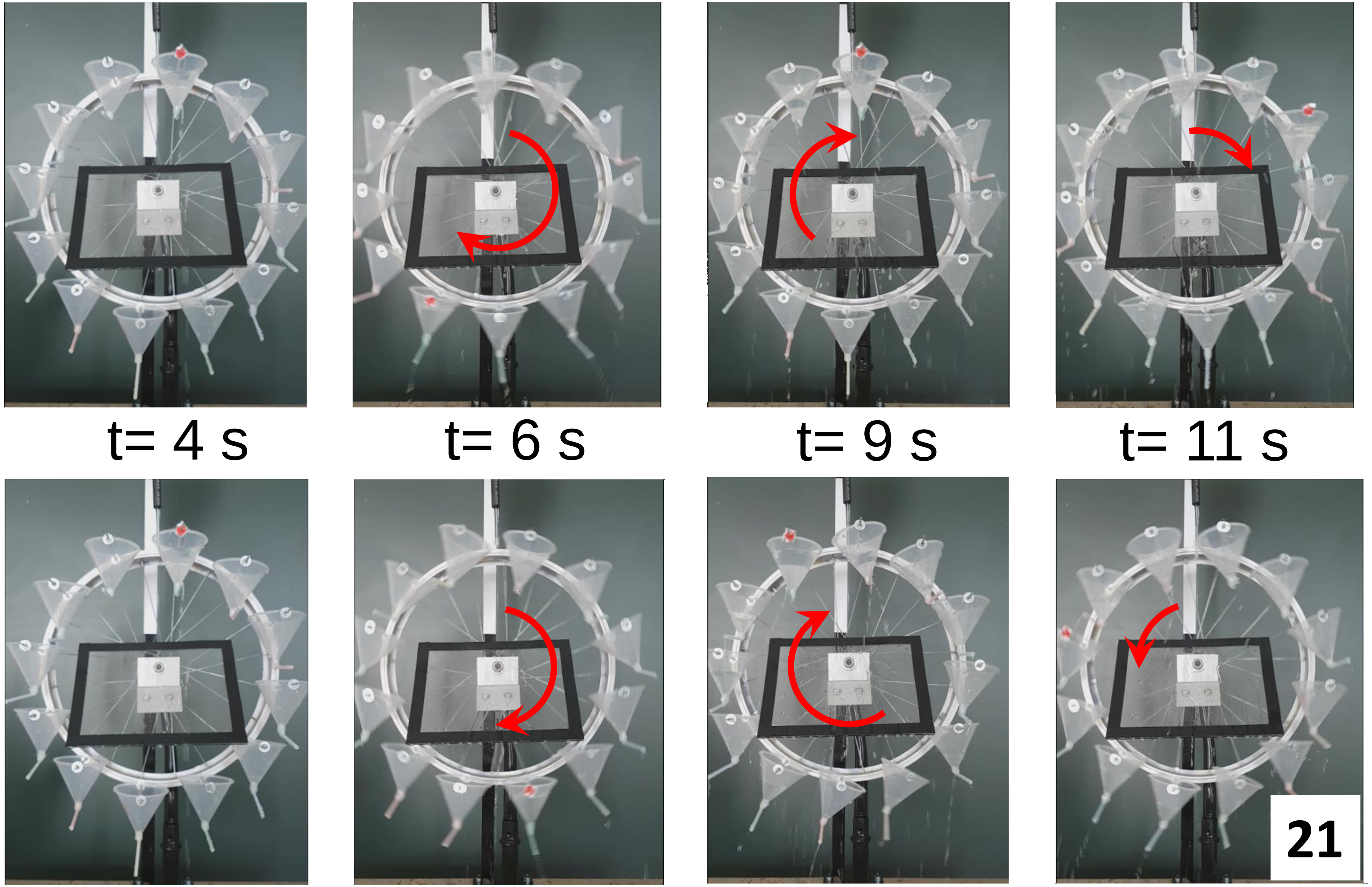
$$x_0 = 0$$
$$y_0 = 1$$
$$\sigma = 10$$
$$\rho = 28$$
$$\beta = 2{,}667$$

$$dt = 0,01$$
$$10000\ points$$

Conditions initiales :

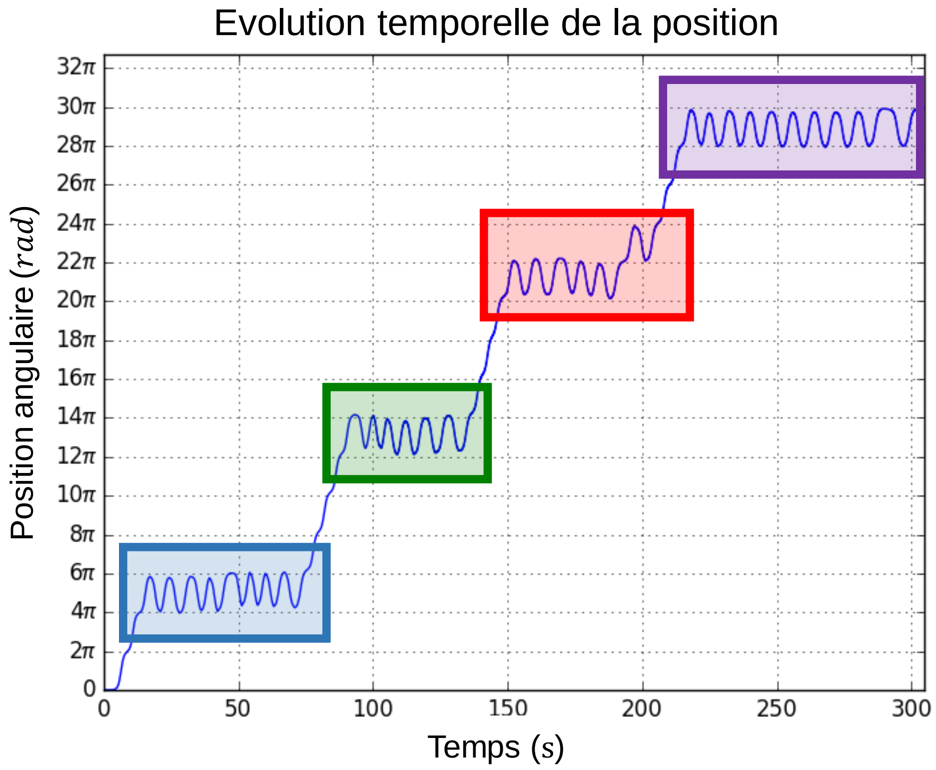


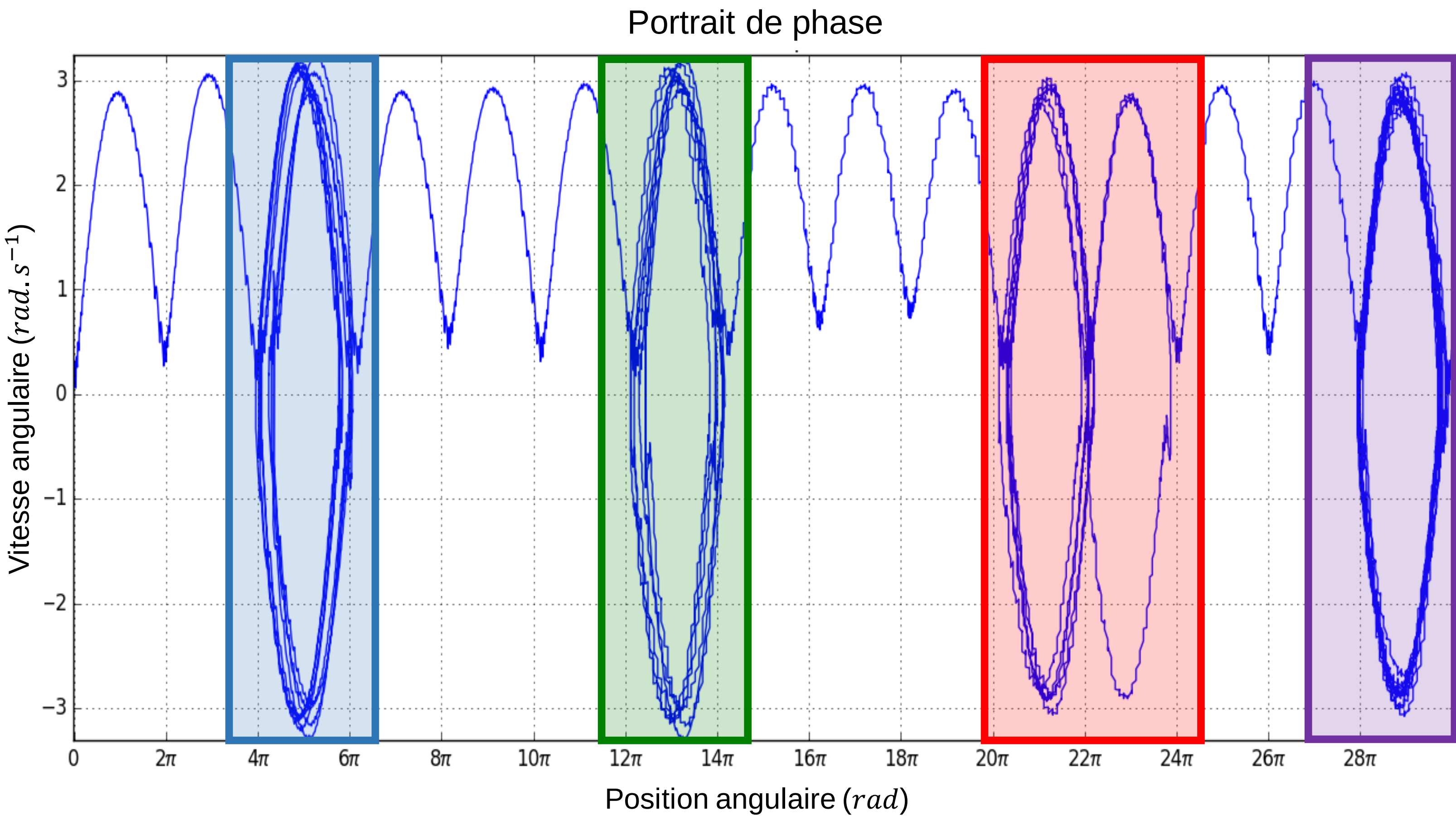




Périodicité avec bifurcation :

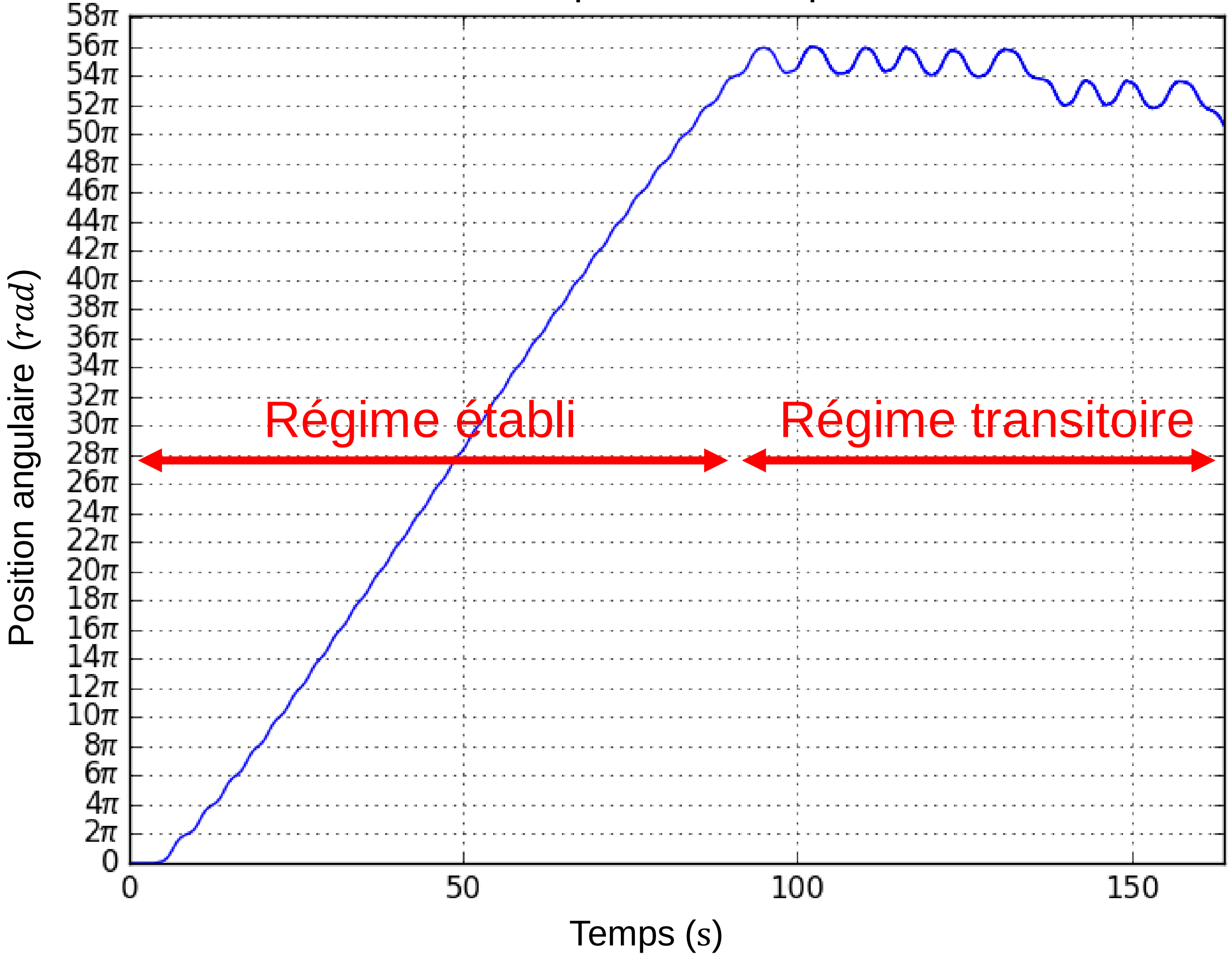
$$\boxed{\begin{gathered}\boldsymbol{\theta_0 = 2{,}37°} \\ \boldsymbol{D_{eau} = 9{,}8 g.s^{-1}}\end{gathered}}$$





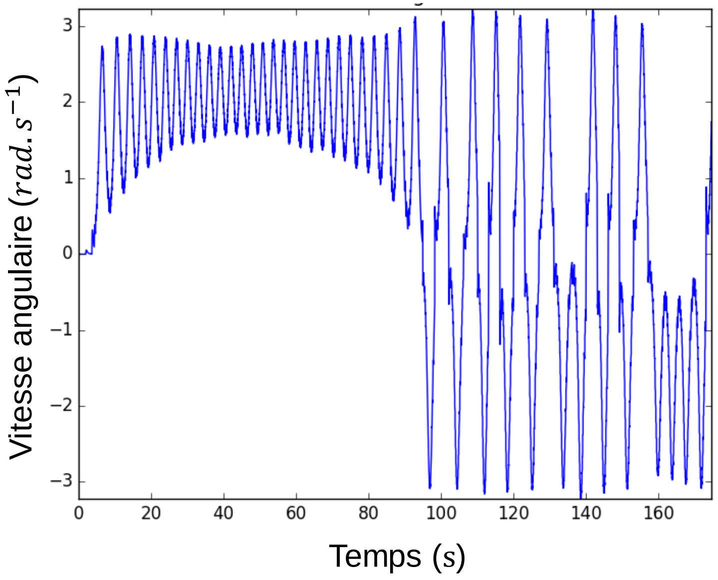
Régime « permanent » puis transitoire :

Evolution temporelle de la position

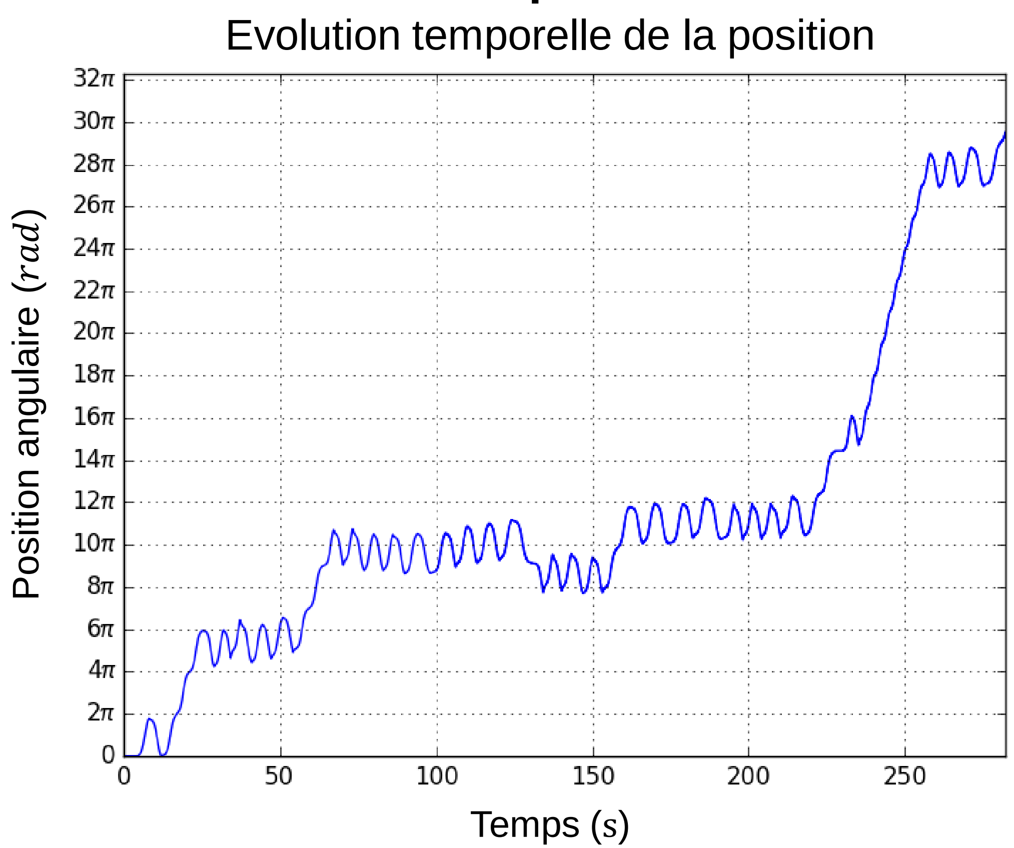


$$\boldsymbol{\theta_0 = 5,1°}$$
$$\boldsymbol{D_{eau} = 5,7 g.s^{-1}}$$

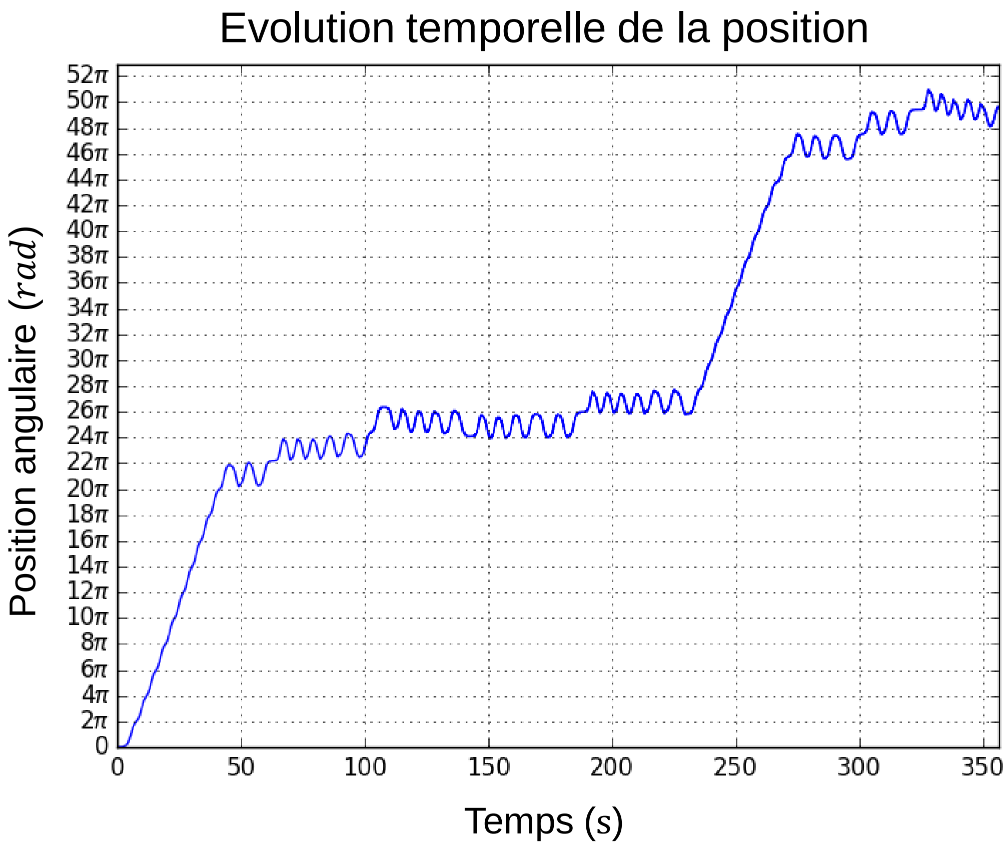
Evolution temporelle de la vitesse



Mouvement chaotiques :



$$\boxed{\begin{array}{c}\theta_0 = 2,63° \\ D_{eau} = 10,4g.s^{-1}\end{array}}$$



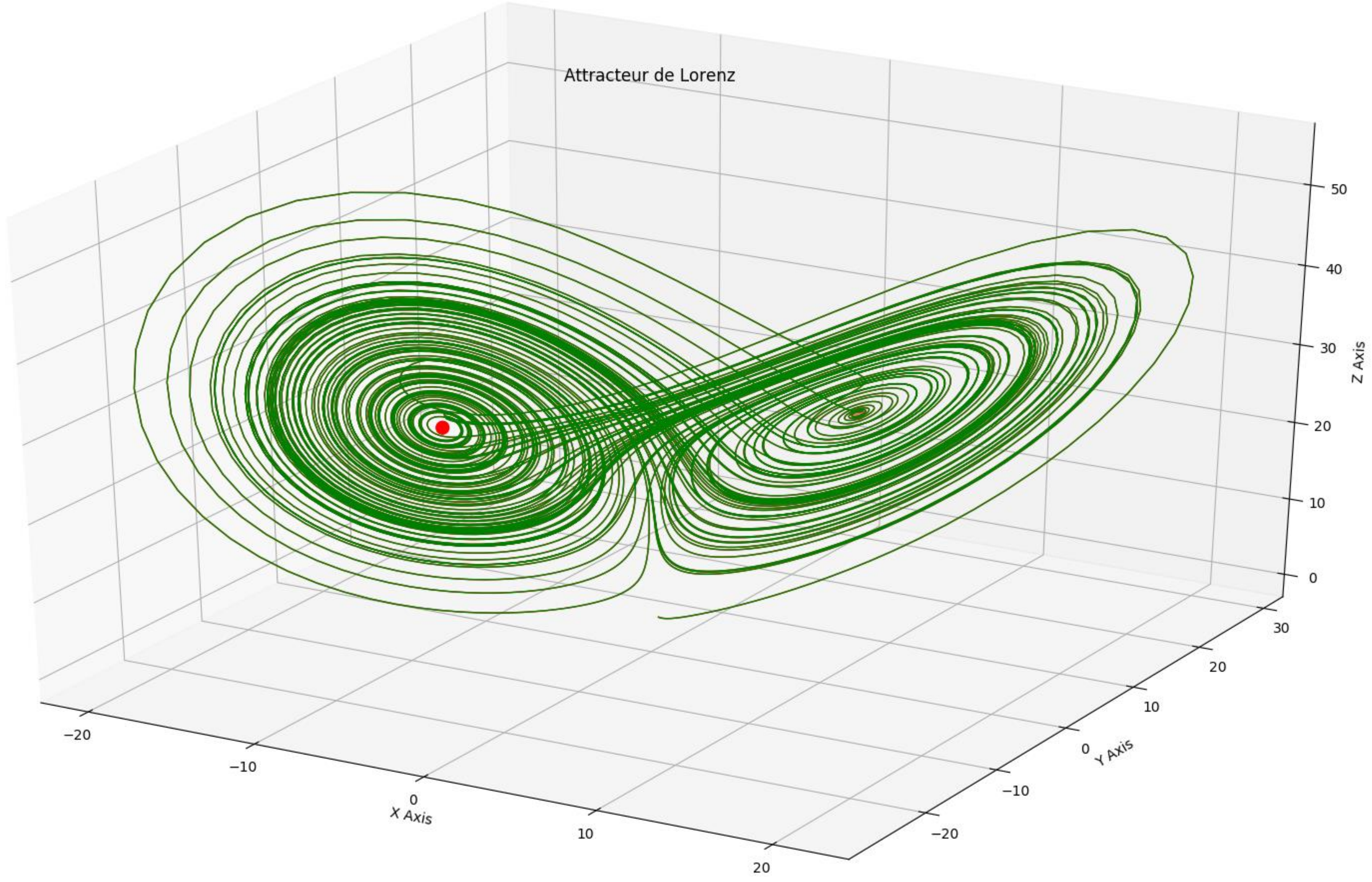
$$\boxed{\begin{array}{c}\theta_0 = 6,62° \\ D_{eau} = 8,2g.s^{-1}\end{array}}$$

L'attracteur :

$x_0 = 0$
$y_0 = 1$
$z_0 = 1,05$

$\sigma = 10$
$\rho = 28$
$\beta = 2,667$

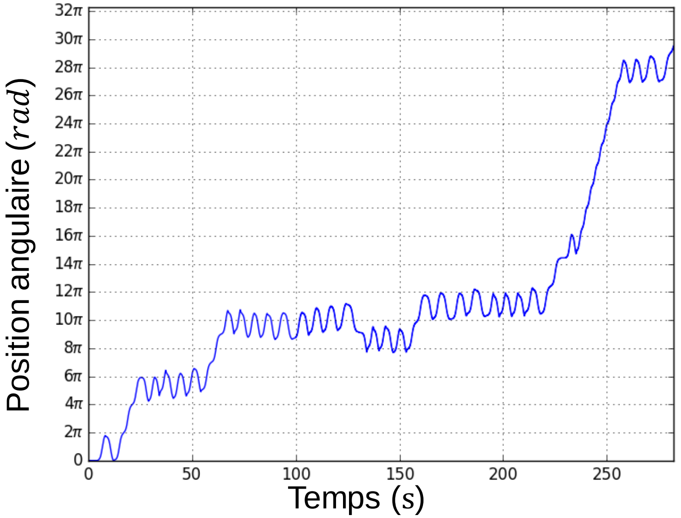
$dt = 0,01$
$10000\ points$



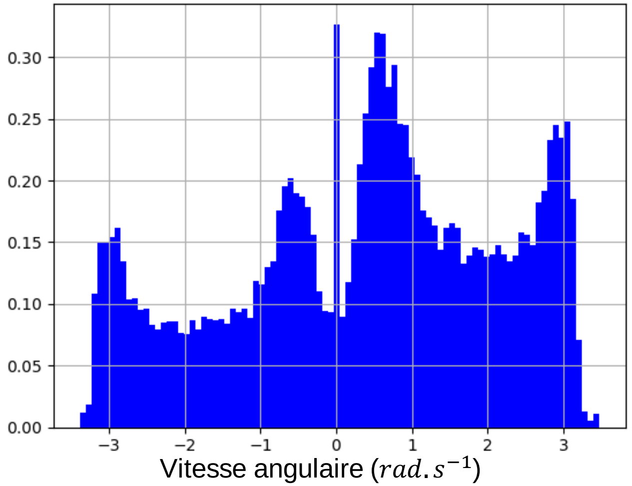
Répartition des vitesses :

$$\boxed{\begin{array}{c}\boldsymbol{\theta_0 = 2,63°} \\ \boldsymbol{D_{eau} = 10,4 g.s^{-1}}\end{array}}$$

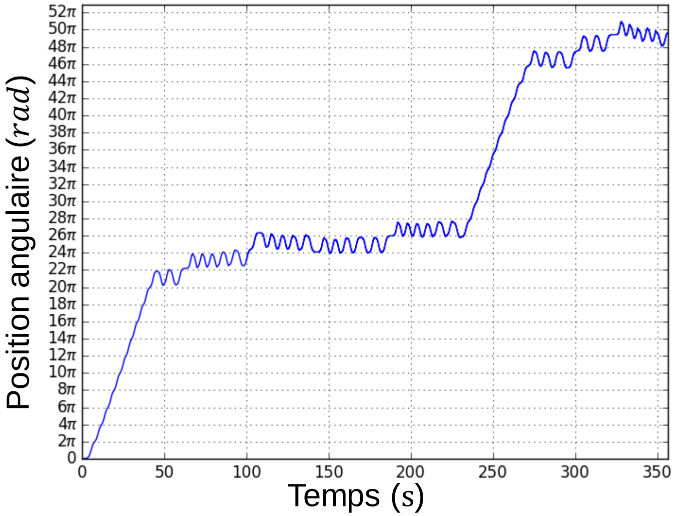
Evolution temporelle de la position

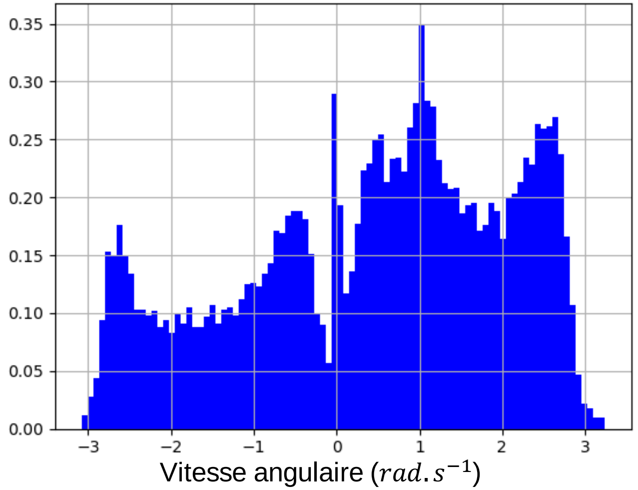


Répartition de la vitesse



$$\boxed{\begin{array}{c}\boldsymbol{\theta_0 = 6,62°} \\ \boldsymbol{D_{eau} = 8,2 g.s^{-1}}\end{array}}$$





Conclusion

Problématique : Comment pouvons-nous appréhender les écarts pour expliquer nos différences de comportements au cours de ces expériences ?

Un système chaotique :

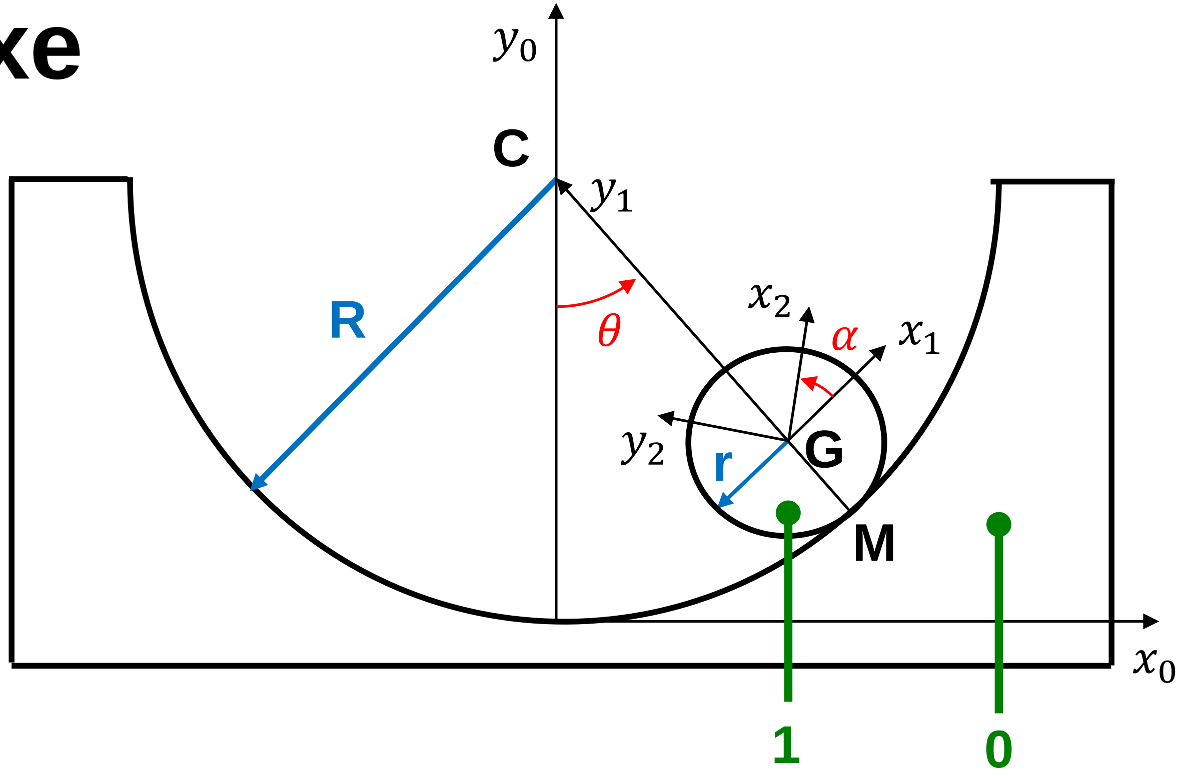
	Expérience	**Simulation numérique**
Un système sensible	2 expériences dans les mêmes conditions	Faible variation des conditions initiales
Bifurcation	Etude de la position angulaire	Mise en évidence avec les chiffres significatifs
Caractère aléatoire	Régime « permanent » puis transitoire	Système non linéaire

De l'ordre dans le chaos :

	Expérience	**Simulation numérique**
« en moyenne »	Répartition des vitesses	L'ensemble des solutions s'accumule sur l'attracteur

Annexe

Démonstration J_{exp} :



Roulement sans glissement en M : $\dot{\alpha} = -\frac{R}{r}\dot{\theta}$

TEC appliqué à **1** : $\frac{d}{dx}E_{c,1/0} = P_{g\to 1/0} + P_{0\to 1/0}$

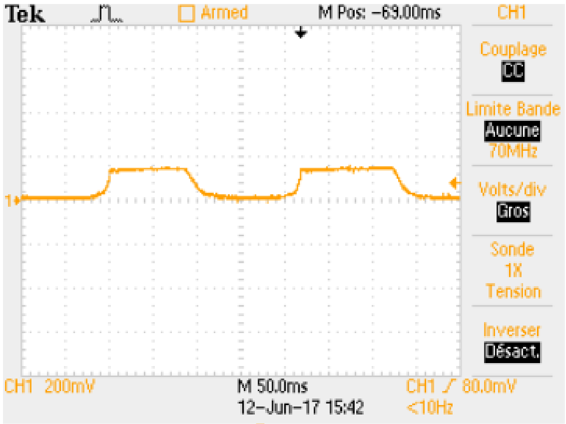
Avec : $E_{c,1/0} = m(R-r)^2\dot{\theta}^2 + J_{exp}\left(1-\frac{R}{r}\right)^2\dot{\theta}^2$

$P_{0\to 1/0} = P_{0\leftrightarrow 1} + P_{1\to 0/0} = 0$

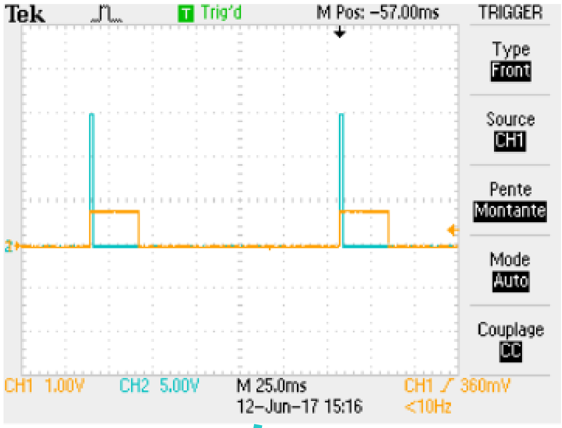
$P_{g\to 1/0} = -mg(R-r)\sin(\theta)\,\dot{\theta}$

Donc, $(J_{exp}+mr^2)\left(1-\frac{R}{r}\right)^2\dot{\theta}\ddot{\theta} = -mg(R-r)\sin(\theta)\,\dot{\theta} \iff \ddot{\theta} + \frac{mgr^2}{(J_{exp}+mr^2)(R-r)}\sin(\theta) = 0$

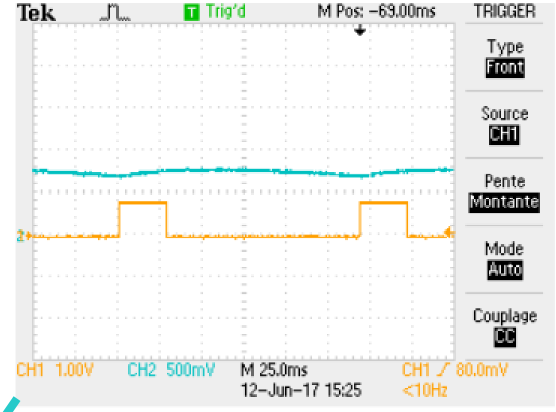
$$\omega_0 = \frac{2\pi}{T} = \sqrt{\frac{mgr^2}{(J_{exp}+mr^2)(R-r)}} \Longrightarrow \boxed{J_{exp} = \frac{T^2}{4\pi^2}\frac{mgr^2}{R-r} - mr^2}$$



Tek
Armed
M Pos: -69.00ms
CH1
Couplage
CC
Limite Bande
Aucune
70MHz
Volts/div
Gros
Sonde
1X
Tension
Inverser
Désact.
CH1 200mV
M 50.0ms
12-Jun-17 15:42
CH1 ∕ 80.0mV
<10Hz

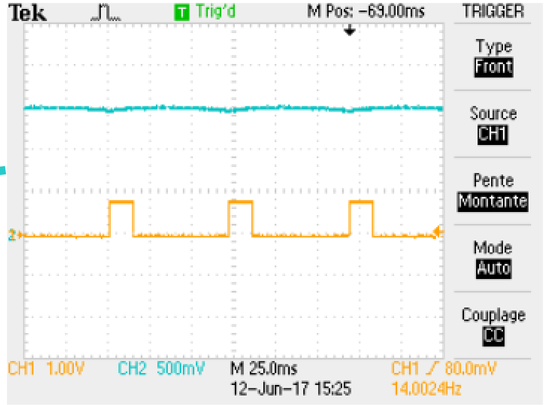


Tek
Trig'd
M Pos: -57.00ms
TRIGGER
Type
Front
Source
CH1
Pente
Montante
Mode
Auto
Couplage
CC
CH1 1.00V
CH2 5.00V
M 25.0ms
12-Jun-17 15:16
CH1 ∕ 360mV
<10Hz



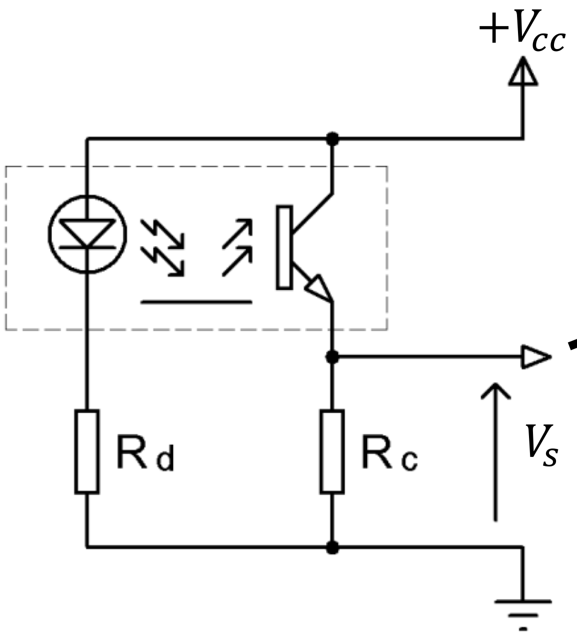
Tek
Trig'd
M Pos: -69.00ms
TRIGGER
Type
Front
Source
CH1
Pente
Montante
Mode
Auto
Couplage
CC
CH1 1.00V
CH2 500mV
M 25.0ms
12-Jun-17 15:25
CH1 ∕ 80.0mV
<10Hz

$f = 7\ Hz$

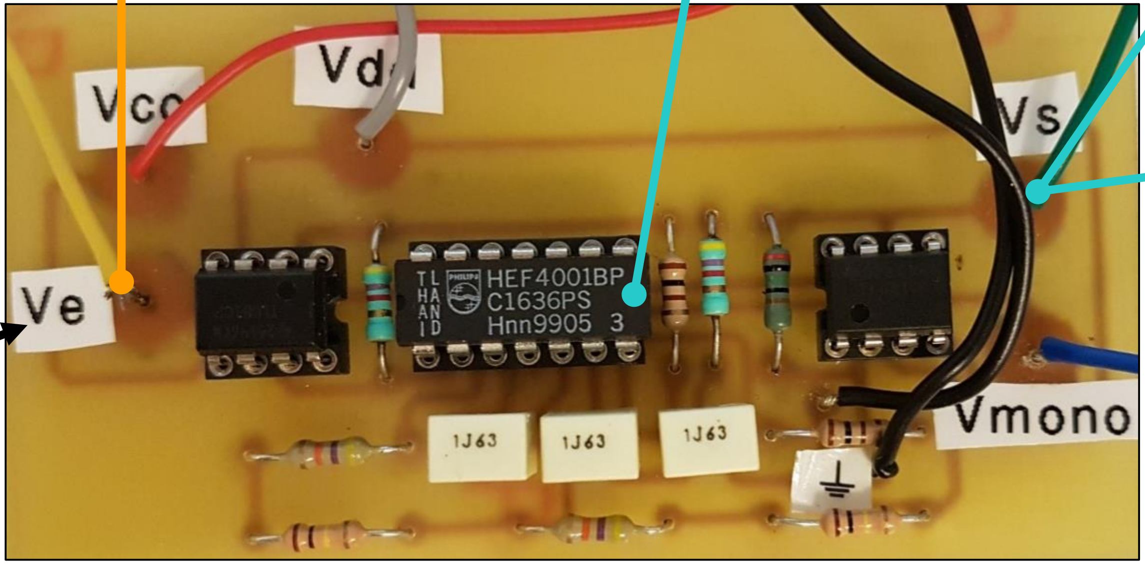


Tek
Trig'd
M Pos: -69.00ms
TRIGGER
Type
Front
Source
CH1
Pente
Montante
Mode
Auto
Couplage
CC
CH1 1.00V
CH2 500mV
M 25.0ms
12-Jun-17 15:25
CH1 ∕ 80.0mV
14.0024Hz

$f = 14\ Hz$



$+V_{cc}$
Rd
Rc
V_s



Vcc
Vd
Ve
Vs
Vmono
HEF4001BP
C1636PS
Hnn9905 3
1J63
1J63
1J63

Annexe

Programme de l’attracteur de Lorenz :

Méthode d’Euler :

```
def Lorenz(x,y,z,s,r,b):
    dx=Decimal(s)*(Decimal(y)-Decimal(x))
    dy=Decimal(r)*Decimal(x)-Decimal(y)-Decimal(x)*Decimal(z)
    dz=Decimal(x)*Decimal(y)-Decimal(b)*Decimal(z)
    return dx,dy,dz
```

Utilisation du module Decimal :

```
        for i in range(nbpts):
            dx,dy,dz=Lorenz(X[i],Y[i],Z[i],s,r,b)
            X[i+1]=float(Decimal(X[i])+Decimal(dx)*Decimal(dt))
            Y[i+1]=float(Decimal(Y[i])+Decimal(dy)*Decimal(dt))
            Z[i+1]=float(Decimal(Z[i])+Decimal(dz)*Decimal(dt))
```

Les points critiques :

```
        a=sqrt(b*(r-1))
        XptsCrit=[-a,a]
        YptsCrit=[-a,a]
        ZptsCrit=[r-1,r-1]
        axe.scatter(XptsCrit,YptsCrit,ZptsCrit,s=80,c='r',marker='o')
```

Annexe

Programme de traitement de données :

Le tableur :

	A	B	C	D	E	F
1	0	-0,00353367	-0,002136	17,14	Durée:	5min04
2	0,025	-0,00605819	-0,003662	21,01		3 Litres
3	0,05	-0,00555362	-0,003357	24,21	Débit (g/s) :	9,8
4	0,075	-0,00555362	-0,003357	28,18	Angle initial :	2,37°
5	0,1	-0,00555362	-0,003357	32,25		
6	0,125	-0,00454447	-0,002747	36,21		
7	0,15	-0,00605819	-0,003662	39,22		
8	0,175	-0,00605819	-0,003662	42,28		
9	0,2	-0,00504905	-0,003052	47,2		
10	0,225	-0,00555362	-0,003357	51,29		

A – Temps
B – Vitesse angulaire
C –Tension de sortie
D –Changement de signe

fx =(C1*2*PI())/(36*0,1055)

Conversion en liste:

```
wb=xlrd.open_workbook('Exp n1.xlsx')
sh=wb.sheet_by_name(u'Feuil1')

acqTps=sh.col_values(0)
acqVit=sh.col_values(1)
chgTps=sh.col_values(3)
```

Annexe

Programme de traitement de données :

Changement de signe :

```
from numpy import inf,pi,arange
```

```
def traitement_Donnees(acqVit,acqTps,chgTps):
    Vit=[]
    b=True
    i=0
    for k in range(0,len(acqVit)):
        if chgTps[i]<=acqTps[k]:
            b=not(b)
            i+=1
        if b:
            Vit.append(acqVit[k])
        else:
            Vit.append(acqVit[k]*(-1))
    return Vit
```

```
for k in range(len(chgTps)):
    if type(chgTps[k])==str:
        chgTps[k]=inf

Vit=traitement_Donnees(acqVit,acqTps,chgTps)
```